MTH 263, Homework #6, Summer 2023 Name KEY

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Complete two interactions of Newton's method by hand for the function $f(x) = x^3 - 3$ with starting condition $x_0 = 1.4$. $X_2 = 1.442251$ See Excel file

2. Use Newton's method (and technology) to final all the zeros of the function to within 0.001 (there may be more than one zero to find).
   a. $f(x) = x^3 + 4$ $X = -1.5874$
   b. $h(x) = x - 2\sqrt{x+1}$ $X = 4.828427$
   c. $a(x) = x^3 - \cos x$ $X = 0.865474$
   d. $g(x) = x^5 + x - 1$ $X = 0.754878$
   e. $y = x - 3 + \ln x$ $X = 2.20794$

   See Excel file

3. Explain why Newton's method might fail. The derivative might be zero which would make the ratio in the formula undefined. There might be no zero. Could fail to converge.

4. Find the indefinite integrals.
   a. $\int x + 7\, dx$
   b. $\int (1 + 3t)t^2 dt$
   c. $\int \sec y\, (\tan y - \sec y) dy$
   d. $\int x - \frac{5}{x} dx$
   e. $\int \sqrt[4]{x^3} + 1\, dx$
   f. $\int t^2 - \cos t\, dt$
   g. $\int 2x - 4^x dx$

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5. Following the example of the proof of the formula for the $\sum_{i=1}^{n} i^3$, complete a similar proof for $\sum_{i=1}^{n} i^2$. [Hint: it's a two-step proof. First show that the proposed formula works for some "base" value of *i*, and second, show that if it works for n, it also works for n+1 (find the sum of n+1 two different ways and show that they are the same).] pg 2

6. Use the limit process to evaluate the following integrals. Show all steps.
   a. $\int_0^1 (x^2 + x) dx$ 5/6
   b. $\int_1^4 (64 - x^3) dx$ 128.25
   c. $\int_{-1}^5 (3x + 4) dx$ 60

   See Excel file

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1a. $\int x+7 \, dx = \frac{1}{2}x^2 + 7x + C$

b. $\int t^2 + 3t^3 \, dt = \frac{1}{3}t^3 + \frac{3}{4}t^4 + C$

c. $\int \sec y \tan y - \sec^2 y \, dy = \sec y - \tan y + C$

d. $\int x - \frac{5}{x} \, dx = \frac{1}{2}x^2 - 5\ln x + C$

e. $\int x^{3/4} + 1 \, dx = \frac{4}{7}x^{7/4} + x + C$

f. $\int t^2 - \cos t \, dt = \frac{1}{3}t^3 - \sin t + C$

g. $\int 2x - 4^x \, dx = x^2 - \frac{4^x}{\ln 4} + C$

5. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

base case: $n=1$ $\quad \sum_{i=1}^{1} i^2 = 1^2 = 1$ and $\frac{1(1+1)(2(1)+1)}{6} = \frac{1(2)(3)}{6} = 1$ ✓

induction case: suppose true for k, prove for k+1

$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$ then $\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2 =$

$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\frac{6}{6}\right] =$

$(k+1)\left[\frac{2k^2+k+6k+6}{6}\right] = \frac{k+1}{6}\left[2k^2+7k+6\right] = \frac{k+1}{6}\left[(2k+3)(k+2)\right] =$

$\frac{(k+1)(k+2)(2k+3)}{6} = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$ which was to be shown.

$\therefore$, the formula works for all n

/QED.