MTH 263, Homework #7, Summer 2023 Name KEY

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Use the Fundamental Theorem of Calculus to evaluate the definite integrals.

   a. $\int_0^1 (x^2+x)\,dx$ $\quad \frac{1}{3}x^3+\frac{1}{2}x^2\Big|_0^1=\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$

   c. $\int_1^4 (64-x^3)\,dx$ $\quad 64x-\frac{1}{4}x^4\Big|_1^4=\frac{513}{4}$

   b. $\int_{-1}^5 (3x+4)\,dx$ $\quad \frac{3}{2}x^2+4x\Big|_{-1}^5=60$

2. Integrate the following integrals. If you use substitution, clearly indicate u and du.

   a. $\int e^{\tan x}\sec^2 x\,dx$

   b. $\int t\sqrt[3]{t-4}\,dt$

   c. $\int_1^5 x^2\sqrt{x-1}\,dx$

   d. $\int_{-\pi/2}^{\pi/2} \sin^2 x\cos x\,dx$

   e. $\int (3-x)7^{(3-x)^2}\,dx$

   f. $\int \left(1+\frac{1}{t}\right)^3\frac{1}{t^2}\,dt$

   g. $\int_{-2}^{2} x(x^2+1)^3\,dx$

   pg 2-3

3. Integrate.

   a. $\int \frac{x(x-2)}{(x-1)^3}\,dx$

   b. $\int \frac{1}{x\ln(x^3)}\,dx$

   c. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$

   d. $\int \frac{\sinh(x)}{1+\sinh^2(x)}\,dx$

   e. $\int \frac{\sec x\tan x}{\sec x-1}\,dx$

   f. $\int_{-2}^{2} \frac{dx}{x^2+4x+13}$

   g. $\int \frac{\arccos(x)}{\sqrt{1-x^2}}\,dx$

   h. $\int \operatorname{sech}^3 x\tanh x\,dx$

   pg 3-4

4. Find the average value of the function $f(x)=\frac{x^2+4}{x}$, on the interval [1,4]. pg 4

5. Find $F'(x)$ if $F(x)=\int_1^{x^2}\frac{1}{t}\,dt$. pg 4

6. Use the definition of $\coth(x)$ to show that the derivative is $-\operatorname{csch}^2 x$. pg 4

2 a. $\int e^{\tan x} \sec^2 x\, dx$

$u = \tan x$
$du = \sec^2 x\, dx$

$\int e^u\, du = e^u + C$

$e^{\tan x} + C$

b. $\int t \sqrt[3]{t-4}\, dt$

$u = \sqrt[3]{t-4}$
$u^3 = t-4$
$u^3 + 4 = t$
$3u^2\, du = dt$

$\int (u^3+4) u \cdot 3u^2\, du =$
$\int 3u^6 + 12u^3\, du =$
$\frac{3}{7}u^7 + 3u^4 + C$

$\frac{3}{7}(t-4)^{7/3} + 3(t-4)^{4/3} + C$

c. $\int_1^5 x^2 \sqrt{x-1}\, dx$

$u = \sqrt{x+1}$
$u^2 - 1 = x$
$2u\, du = dx$

$\int (u^2-1)^2 \cdot u \cdot 2u\, du =$
$\int (u^4 - 2u^2 + 1)\, 2u^2\, du =$
$\int 2u^6 - 4u^4 + 2u^2\, du =$
$\frac{2}{7}u^7 - \frac{4}{5}u^5 + \frac{2}{3}u^3 + C$

$\frac{2}{7}(x-1)^{7/2} - \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C \Big|_1^5$

$\frac{2}{7}(5-1)^{7/2} - \frac{4}{5}(5-1)^{5/2} + \frac{2}{3}(5-1)^{3/2} - \cancel{\frac{2}{7}(1-1)^{7/2}}^{0} + \cancel{\frac{4}{5}(1-1)^{5/2}}^{0} - \cancel{\frac{2}{3}(1-1)^{3/2}}^{0}$

$= \frac{2}{7}(2^7) - \frac{4}{5}(2)^5 + \frac{2}{3}(2)^3 = \frac{1712}{105}$

d. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\, dx$

$u = \sin x$
$du = \cos x\, dx$

$\int u^2\, du = \frac{1}{3}u^3 + C$

$\frac{1}{3}\sin^3 x \Big|_{-\pi/2}^{\pi/2} = \frac{1}{3}(1)^3 - \frac{1}{3}(-1)^3 = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

e. $\int (3-x)\, 7^{(3-x)^2}\, dx$

$u = (3-x)^2$
$du = -2(3-x)\, dx$
$-\frac{1}{2}\, du = (3-x)\, dx$

$\int -\frac{1}{2} 7^u\, du$
$-\frac{1}{2} \cdot \frac{1}{\ln 7} 7^u + C$

$\frac{7^{(3-x)^2}}{2\ln 7} + C$

f. $\int \left(1 + \frac{1}{t}\right)^3 \cdot \frac{1}{t^2}\, dt$

$u = 1 + \frac{1}{t}$
$du = -\frac{1}{t^2}\, dt$
$-du = \frac{1}{t^2}\, dt$

$\int -u^3\, du$
$-\frac{1}{4}u^4 + C$

$-\frac{1}{4}\left(1 + \frac{1}{t}\right)^4 + C$

1g. $\int_{-2}^{2} x(x^2+1)^3\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int \frac{1}{2}u^3\,du =$

$\frac{1}{2}\cdot\frac{1}{4}u^4 + C$

$\frac{1}{8}(x^2+1)^4 \Big|_{-2}^{2} =$

$\frac{1}{8}(4+1)^4 - \frac{1}{8}(4+1)^4 = 0$

2a. $\int \frac{x(x-2)}{(x-1)^3}\,dx$

$u = x-1$

$du = dx$

$u+1 = x$

$u-1 = x-2$

$\int \frac{(u+1)(u-1)}{u^3}\,du =$

$\int \frac{u^2-1}{u^3}\,du = \int \frac{1}{u} - u^{-3}\,du =$

$\ln u - \frac{1}{2}u^{-2} + C$

$\ln|x-1| - \frac{1}{2(x-1)^2} + C$

b. $\int \frac{1}{x\ln(x^3)}\,dx = \int \frac{1}{x\cdot 3\ln x}\,dx = \frac{1}{3}\int \frac{1}{x\ln x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\frac{1}{3}\int \frac{1}{u}\cdot du = \frac{1}{3}\ln u + C$

$\frac{1}{3}\ln|\ln x| + C$

c. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx = \int \frac{1}{(x-1)\sqrt{x^2-2x+1-1}}\,dx = \int \frac{1}{(x-1)\sqrt{(x-1)^2-1}}\,dx = \operatorname{arcsec}(x-1) + C$

d. $\int \frac{\sinh x}{1+\sinh^2 x}\,dx = \int \frac{\sinh x}{\cosh^2 x}\,dx$

$u = \cosh x$

$du = \sinh x\,dx$

$\int \frac{1}{u^2}\,du = -\frac{1}{u} + C$

$\frac{-1}{\cosh x} + C = -\operatorname{sech} x + C$

e. $\int \frac{\sec x\tan x}{\sec x - 1}\,dx$

$u = \sec x - 1$

$du = \sec x\tan x\,dx$

$\int \frac{1}{u}\,du = \ln u + C$

$\ln|\sec x - 1| + C$

f. $\int_{-2}^{2} \frac{dx}{x^2+4x+13} = \int_{-2}^{2} \frac{dx}{x^2+4x+4+9} = \int_{-2}^{2} \frac{dx}{(x+2)^2+3^2} = \frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C\Big|_{-2}^{2} =$

$\frac{1}{3}\arctan\left(\frac{4}{3}\right)$

g. $\int \frac{\arccos x}{\sqrt{1-x^2}}\,dx$

$u = \arccos x$

$du = -\frac{1}{\sqrt{1-x^2}}\,dx$

$-du = \frac{1}{\sqrt{1-x^2}}\,dx$

$\int -u\,du = -\frac{1}{2}u^2 + C$

$-\frac{1}{2}(\arccos x)^2 + C$

3h. $\int \operatorname{sech}^3 x \tanh x \, dx = \int \operatorname{sech}^2 x (\operatorname{sech} x \tanh x)\, dx$ $u = \operatorname{sech} x$

$-du = \cancel{+}\operatorname{sech} x \tanh x \, dx$

$\int -u^2 \, du = -\frac{1}{3}u^3 + C$

$-\frac{1}{3}\operatorname{sech}^3 x + C$

4. $\frac{1}{4-1}\int_1^4 \frac{x^2+4}{x}\, dx = \frac{1}{3}\int_1^4 x + \frac{4}{x}\, dx = \frac{1}{3}\left[\frac{1}{2}x^2 + 4\ln x\right|_1^4 = \frac{1}{3}\left[8 + 4\ln 4 - \frac{1}{2} - 0\right]$

$\frac{1}{3}\left[\frac{15}{2} + 4\ln 4\right]$

5. $\frac{d}{dx}\left[\int_1^{x^2} \frac{1}{t}\, dt\right] = \frac{1}{x^2}\cdot 2x = \frac{2}{x}$

6. $\coth x = \frac{\cosh x}{\sinh x}$

$\frac{d}{dx}[\coth x] = \frac{d}{dx}\left[\frac{\cosh x}{\sinh x}\right] = \frac{\sinh x \cdot \sinh x - \cosh x \cosh x}{\sinh^2 x} = \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x} = \frac{-1}{\sinh^2 x} =$

$-\operatorname{csch}^2 x$ /QED