MTH 263, Homework #8, Summer 2023 Name KEY

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the area bounded by the curves.
   a. $f(x) = x^4 - 4x^2$ and $g(x) = x^3 - 4x$ pg 2
   b. $x = y^4, y = \sqrt{2x}, y = 0$

2. Calculate the volumes of the solids of revolution as indicated below. Calculate the volume both with the shell method and with the disk or washer method and verify that the volume is the same in each case.
   a. $y = 2x^2, y = 0, x = 2$ Revolved around the:
      i. y-axis
      ii. x-axis
      iii. the line $y = 8$ pg 2
      iv. the line $x = 2$
   b. $y = \frac{10}{x^2}, y = 0, x = 1, x = 5$
      i. y-axis
      ii. x-axis
      iii. $y = 10$ pg 3
      iv. $x = 5$

3. Find the surface area when the function $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $1 \leq x \leq 2$ is revolved around the x-axis. pg 3

4. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2-x}$ and the x-axis, around the x-axis, where x and y are measured in meters. Find the tank's volume. pg 3

5. An ornamental light bulb is designed by resolving the graph of $y = \frac{1}{3}x^{1/2} - x^{3/2}$ on the interval [0,1/3] around the x-axis, where x and y are measured in feet. Find the surface area of the bulb and use the result to approximate the amount of glass needed to make the bulb (assume that the glass is 0.015 inches thick). pg 3

6. Find the length of arc in rectangular coordinates. You may need to calculate the value numerically.
   a. $y = 1 + 6x^{\frac{3}{2}}, [0,1]$
   b. $y = (1 - e^{-x}), [0,2]$
   c. $y = \ln(\sec x), \left[0, \frac{\pi}{4}\right]$
   d. $y = \sqrt[3]{x}, [0,1]$

   pg 3-4

7. Integrate.

a. $\int \tan(x)\ln(\cos x)dx$

b. $\int \frac{1}{3t+1}-\frac{17}{(4t-1)^2}dt$

c. $\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}dx$

d. $\int \frac{5}{3e^x-2}dx$

Pg 4

1a. $x^4-4x^2 = x^3-4x$

$x^4-x^3-4x^2+4x=0$

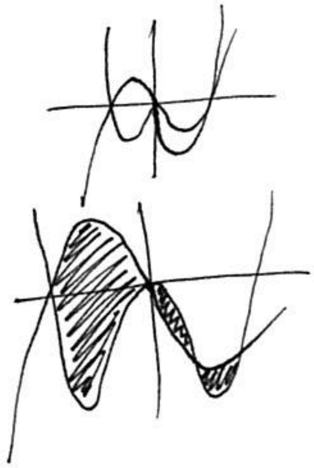
$x\left(x^3-x^2-4x+4\right)=0$

$x\left[(x^2)(x-1)-4(x-1)\right]=0$

$x\left[(x-1)(x^2-4)\right]=0$

$x(x-1)(x-2)(x+2)=0$

$x=0,1,2,-2$



$$\int_{-2}^{0}(x^3-4x)-(x^4-4x^2)\,dx+\int_0^1 x^4-4x^2-(x^3-4x)\,dx+\int_1^2(x^3-4x)-(x^4-4x^2)\,dx=$$

$$\int_{-2}^{0}x^3-4x-x^4+4x^2\,dx+\int_0^1 x^4-4x^2-x^3+4x\,dx+\int_1^2 x^3-4x-x^4+4x^2\,dx=$$

$$\left[\tfrac14x^4-2x^2-\tfrac15x^5+\tfrac43x^3\right|_{-2}^{0}+\left[\tfrac15x^5-\tfrac43x^3-\tfrac14x^4+2x^2\right|_0^1+\left[\tfrac14x^4-2x^2-\tfrac15x^5+\tfrac43x^3\right|_1^2=$$

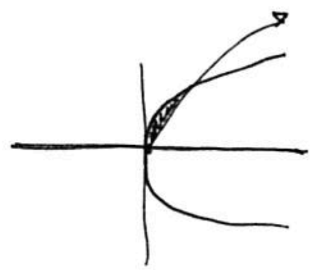
$$124/15 \quad + \quad \frac{37}{60} \quad + \quad 53/60 = \frac{293}{30}$$

b. $x=y^4$, $y=\sqrt{2x}$, $y=0$

$y^2=2x = \frac12y^2=x$

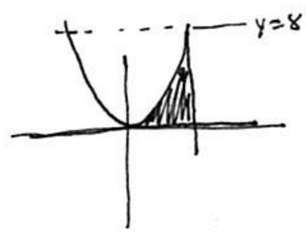
intersections at $y=\frac{1}{\sqrt2}$, $y=0$

or $x=0$, $x=\frac14$



$$\int_0^{1/\sqrt2}\tfrac12y^2-y^4\,dy = \tfrac16y^3-\tfrac15y^5\Big|_0^{1/\sqrt2} = \frac{1}{30\sqrt2}$$

2a. $2(2)^2=8$



i. y-axis is $2\pi\int_0^2 x\underbrace{(2x^2)}_{2x^3}\,dx = [2\pi]\left(\tfrac12x^4\right|_0^2 = 2\pi\cdot 8=16\pi$

ii. x-axis is $\pi\int_0^2\underbrace{(2x^2)^2}_{4x^4}\,dx = \pi\left[\tfrac45x^5\right|_0^2 = 128/5\,\pi$

if you use shell or washer method, you should get the same answer either way.

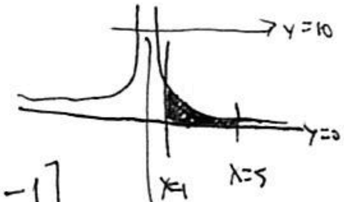
iii. $y=8$

$$\pi\int_0^2 8^2-(8-2x^2)^2\,dx=\pi\int_0^2 \cancel{64}-\cancel{64}+32x^2-4x^4\,dx = \pi\left[\tfrac{32}{3}x^3-\tfrac45x^5\right|_0^2 = 896\pi/15$$

iv. $x=2$ $\quad 2\pi\int_0^2(2-x)(2x^2)\,dx = 2\pi\int_0^2 4x^2-2x^3\,dx = 2\pi\cdot\frac83=\frac{16\pi}{3}$

2b. i. y-axis

$2\pi\int_1^5 x\left(\frac{10}{x^2}\right)dx = 2\pi\int_1^5 \frac{10}{x}\,dx = 2\pi\left[10\ln x\right|_1^5 = 2\pi\left[10\ln 5\right]$



ii. x-axis

$\pi\int_1^5\left(\frac{10}{x^2}\right)^2dx = \pi\int_1^5\frac{100}{x^4}\,dx \quad \frac{\pi\, 100}{-3}x^{-3}\Big|_1^5 = -\frac{100\pi}{3}\left[\frac{1}{5^3}-1\right]$

$= 496/15\pi$

iii. y=10

$\pi\int_1^5 (10)^2-\left(10-\frac{10}{x^2}\right)^2dx = \pi\int_1^5 \cancel{100}-\cancel{100}+\frac{200}{x^2}-\frac{100}{x^4}\,dx = \pi\left[-200x^{-1}+\frac{100}{3}x^{-3}\right|_1^5 = 1904/15$

x=5

iv. $2\pi\int_1^5 (5-x)\left(\frac{10}{x^2}\right)dx = 2\pi\int_1^5\frac{50}{x^2}-\frac{50}{x}\,dx = 2\pi\left[-\frac{50}{x}-50\ln x\right|_1^5 = 2\pi\left[-\frac{50}{5}-50\ln 5+\frac{50}{1}+0\right]$

$= 2\pi\left[40-50\ln 5\right]$

3. $2\pi\int_1^2\left(\frac{x^3}{6}+\frac{1}{2x}\right)\sqrt{1+\left(\frac12x^2-\frac{1}{2x^2}\right)^2}\,dx =$

$y' = \frac12x^2-\frac{1}{2x^2}$

$2\pi\int_1^2\left(\frac{x^3}{6}+\frac{1}{2x}\right)\sqrt{1+\frac14x^4-\frac12+\frac{1}{4x^4}}\,dx =$

$(y')^2 = \frac14x^4-\frac12+\frac{1}{4x^4}$

$2\pi\int_1^2\left(\frac{x^3}{6}+\frac{1}{2x}\right)\sqrt{\frac14x^4+\frac12+\frac{1}{4x^4}}\,dx = 2\pi\int_1^2\left(\frac{x^3}{6}+\frac{1}{2x}\right)\sqrt{\left(\frac12x^2+\frac{1}{2x^2}\right)^2}\,dx =$

$2\pi\int_1^2\left(\frac{x^3}{6}+\frac{1}{2x}\right)\left(\frac12x^2+\frac{1}{2x^2}\right)dx = 2\pi\int_1^2\frac{1}{12}x^5+\frac{1}{12}x+\frac14x+\frac{1}{4x^3}\,dx = \pi\int_1^2\frac{1}{12}x^5+\frac13x+\frac14x^{-3}\,dx$

$= 2\pi\left[\frac{1}{72}x^6+\frac16x^2-\frac18x^{-2}\right|_1^2 = 2\left(\frac{47\pi}{32}\right) = \frac{47\pi}{16}$

4. $\pi\int_0^2\left(\frac18x^2\sqrt{2-x}\right)^2dx = \pi\int_0^2\frac{1}{64}x^4(2-x)\,dx = \frac{\pi}{64}\int_0^2 2x^4-x^5\,dx = \frac{\pi}{64}\left[\frac25x^5-\frac16x^6\right|_0^2 =$

$\frac{\pi}{64}\cdot\frac{32}{15} = \frac{\pi}{30}$

5. $2\pi\int_0^{1/3}\left(\frac13x^{1/2}-x^{3/2}\right)\sqrt{1+\frac{1}{36}x^{-1}-\frac12+\frac94x}\,dx =$

$y' = \frac16x^{-1/2}-\frac32x^{1/2}$

$2\pi\int_0^{1/3}\left(\frac13x^{1/2}-x^{3/2}\right)\sqrt{\frac{1}{36}x^{-1}+\frac12+\frac94x}\,dx =$

$(y')^2 = \frac{1}{36}x^{-1}-\frac{3}{6}\cdot\frac{3}{2}+\frac94x$

$\frac12$

$2\pi\int_0^{1/3}\left(\frac13x^{1/2}-x^{3/2}\right)\left(\frac16x^{-1/2}+\frac32x^{1/2}\right)dx =$

$2\pi\int_0^{1/3}\left(\frac{1}{18}+\frac12x-\frac16x-\frac32x^2\right)dx = 2\pi\int_0^{1/3}\frac{1}{18}+\frac13x-\frac32x^2\,dx = 2\pi\left[\frac{1}{18}x+\frac16x^2-\frac12x^3\right|_0^{1/3} = 2\pi\left(\frac{1}{54}\right) = \frac{\pi}{27}$

= SA

volume of glass needed approx $\frac{\pi}{27}\times(0.015) \approx 0.001745$ inches cubed.

6a. $y' = 6\cdot\frac32x^{1/2} = 9x^{1/2}$

$s = \int_0^1\sqrt{1+81x}\,dx$

$u = 1+81x$
$du = 81\,dx$
$\frac{1}{81}du = dx$

$\int u^{1/2}\cdot\frac{1}{81}\,du$

$= \left[\frac{1}{81}\frac23u^{3/2}\right.$

$\left[\frac{1}{81}\cdot\frac23(1+81x)^{3/2}\right|_0^1 = \frac{2}{243}\left[82^{3/2}-1\right]$

6b. $y' = e^x \qquad s = \int_0^2 \sqrt{1+e^{2x}}\, dx \approx 6.7886512$

c. $y' = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x \qquad s = \int_0^{\pi/4} \sqrt{1+\tan^2 x}\, dx = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/4} \sec x\, dx =$

$\ln|\sec x + \tan x| \Big|_0^{\pi/4} = \ln|\sec \pi/4 + \tan \pi/4| - \ln|\sec 0 + \tan 0| =$

$\ln|\sqrt{2}+1| - \ln|1| = \ln|\sqrt{2}+1|$

d. $y' = \frac{1}{3}x^{-2/3}$

$s = \int_0^1 \sqrt{1+\frac{1}{9}x^{-4/3}}\, dx \approx 1.54787$

note, the integral is improper at $x=0$, so your calculator may not converge to a solution

7a. $\int \tan x \ln(\cos x)\, dx \qquad u = \ln(\cos x)$

$du = -\tan x\, dx$

$\int -u\, du = -\frac{1}{2}u^2 + C$

$-\frac{1}{2}(\ln(\cos x))^2 + C$

b. $\int \frac{1}{3t+1} - \frac{17}{(4t-1)^2}\, dt = \frac{1}{3}\ln|3t+1| + \frac{17}{4}(4t-1)^{-1} + C = \frac{1}{3}\ln|3t+1| + \frac{17}{4(4t-1)} + C$

c. $\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}\, dx = \int \frac{1}{(x-1)\sqrt{4x^2-8x+4-1}}\, dx = \int \frac{1}{(x-1)\sqrt{[2(x-1)]^2-1}}\, dx$

$= \int \frac{2}{2(x-1)\sqrt{[2(x-1)]^2-1}}\, dx = \operatorname{arcsec}(2(x-1)) + C$

d. $\int \frac{5}{3e^x-2}\, dx = \int \frac{5e^{-x}}{3-2e^{-x}}\, dx \qquad u = 3-2e^{-x}$

$* \frac{e^{-x}}{e^{-x}} \qquad du = 2e^{-x}dx$

$\frac{1}{2}du = e^{-x}dx$

$\int \frac{5/2}{u}\, du = \frac{5}{2}\ln|u| + C = \frac{5}{2}\ln|3-2e^{-x}| + C$