8/3/2023

Log/Exponential Integrals
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Review for Final Exam

Log/Exponential Integrals

In the past, we've defined the natural log function as the inverse of the exponential function with base e. We alternatively define the natural log function in terms of an accumulation:

$$\ln(x) = \int_1^x \frac{1}{t}\,dt$$

Log Properties

$$\ln e = 1$$
$$\log_a a = 1$$
$$\ln 1 = 0$$
$$\ln(ab) = \ln a + \ln b$$
$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$
$$\ln(a^r) = r \ln a$$
$$\log_a b = \frac{\ln b}{\ln a}$$

Exponential Properties

$$a^m a^n = a^{m+n}$$
$$(a^n)^m = a^{nm}$$
$$\frac{1}{a} = a^{-1}$$
$$\frac{a^n}{a^m} = a^{n-m}$$
$$a^x = e^{x \ln a}$$

For example:

$$\frac{d}{dx}\left[\log \sqrt[3]{x^2+7}\right] = \frac{d}{dx}\left[\frac{1}{3}\log(x^2+7)\right] = \frac{1}{3}\left(\frac{1}{\ln 10}\right)\left(\frac{1}{x^2+7}\right)(2x)$$

For integration, think about problems that might yield logs…

1) All of them are fractions (ratio of functions)
2) Check if the numerator is the derivative of the denominator (or a multiple of it)… the power/function of x must match

3) You may need to do long division: if the numerator is the same degree or higher than the denominator
4) Also have to think about whether the denominator is a sum of squares… because this might not be a log rule at all, it might be in an inverse tangent

Exponential Growth and Decay

$$P(t) = P_0 e^{kt}$$

If k is positive, exponential growth
If k is negative, exponential decay

Related to continuous compounding, where k is the rate of growth (annually)

If the problem asks for the rate of growth/decay at a particular time… then you'll need to find the derivate and plug in the value for time.

If they give you the rate and ask for the accumulation growth/decay, then integrate.

Newton's Law of Cooling.
Is basically exponential growth/decay but with a horizontal asymptote which is not zero.

Hyperbolic Trigonometric Functions

Review for the Final Exam