MTH 263, Quiz #7, Summer 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find all the critical points of the function $f(x) = x^3 - 12x$. Use this information to create a sign chart and sketch the graph.

$$f'(x) = 3x^2 - 12 = 3(x^2-4) = 0 \qquad x = \pm 2$$
$$3(x-2)(x+2)$$

+ − +

−2 2

2. Find the absolute maximum and absolute minimum of the function $f(x) = x^2 + \frac{2}{x}$ on the interval $[1,4]$.

$$f'(x) = 2x - 2x^{-2} = 2x - \frac{2}{x^2} = 0 \qquad 2x^3 - 2 = 0$$
$$2(x^3-1) = 0$$
$$x = 1 \text{ critical point}$$

$f(1) = 1^2 + \frac{2}{1} = 3$ min

$f(4) = 4^2 + \frac{2}{4} = 16 + \frac{1}{2} = \frac{33}{2}$ max

3. Given $f(x) = x^3 - 4x^2 + x + 3$, find any critical points, any intervals where the graph is increasing and where it is decreasing, find any points of inflection, and intervals where the graph is concave up and concave down.

$$f'(x) = 3x^2 - 8x + 1 = 0 \qquad x = \frac{8 \pm \sqrt{64-4(3)(1)}}{2(3)} = \frac{8 \pm \sqrt{52}}{6} = \frac{8 \pm 2\sqrt{13}}{6} = \frac{4 \pm \sqrt{13}}{3}$$

$64 - 12$

$$f''(x) = 6x - 8 = 0$$

$x = 8/6 = \frac{4}{3}$ inflection pt.

+ − +

$\frac{4-\sqrt{13}}{3}$ $\frac{4+\sqrt{13}}{3}$

− +

$4/3$

increasing $\left(-\infty, \frac{4-\sqrt{13}}{3}\right) \cup \left(\frac{4+\sqrt{13}}{3}, \infty\right)$

decreasing $\left(\frac{4-\sqrt{13}}{3}, \frac{4+\sqrt{13}}{3}\right)$

Concave up $(4/3, \infty)$

Concave down $(-\infty, 4/3)$

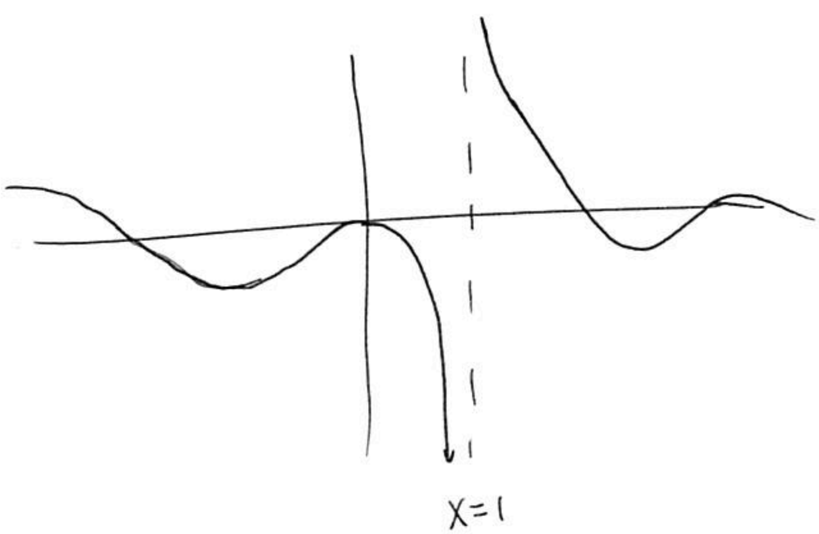
4. Find any horizontal and vertical asymptotes to the graph $f(x) = \frac{x\sin x}{x-1}$. Sketch the graph.

vertical asymptote $X - 1 = 0 \rightarrow X = 1$

horizontal asymptotes

none

$$\begin{array}{r} 1 \\ X-1 \enclose{longdiv}{X} \\ \underline{-X+1} \\ 1 \end{array} \qquad \frac{1}{X-1}$$



$X = 1$