MTH 263, Quiz #8, Summer 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Where is the parabola $y = x^2 - 3$, closest to the point (1,5)?

$$d = \sqrt{(x-1)^2 + (y-5)^2}$$

$$= \sqrt{(x-1)^2 + (x^2-3-5)^2} = \sqrt{(x-1)^2 + (x^2-8)^2} =$$

$$\sqrt{x^2-2x+1 + x^4 - 16x^2 + 64} = \sqrt{x^4 - 15x^2 - 2x + 65}$$

$$d' = \frac{1}{2}(x^4 - 15x^2 - 2x + 65)^{-1/2}(4x^3 - 30x - 2) = 0$$

solve numerically

rel min $x = -2.70465$ $d(-2.7) \approx 3.7674$

rel max $x = -0.0667062$ $d(-0.066) \approx 8.0664$

rel min $x = 2.7713565$ $d(2.77) \approx 1.8$ ← absolute min

2. Evaluate each limit. You may need to perform algebra in order to apply L'Hopital's Rule.

a. $\lim\limits_{x \to 0} x^2 \ln x$ $= \lim\limits_{x \to 0} \frac{\ln x}{x^{-2}} = \lim\limits_{x \to 0} \frac{\frac{1}{x}}{-2x^{-3}} = \lim\limits_{x \to 0} -\frac{1}{2}\left(\frac{1}{x} \cdot x^3\right) = \lim\limits_{x \to 0} -\frac{1}{2}x^2 = 0$

$0 \cdot \infty$ $\quad \frac{\infty}{\infty}$

b. $\lim\limits_{x \to \pi} \frac{1+\cos x}{\sin x}$ $\quad \frac{1-1}{0} = \frac{0}{0}$

$$\lim_{x \to \pi} \frac{-\sin x}{\cos x} = \frac{-0}{-1} = 0$$

3. Use Newton's method to find the real roots of $f(x) = x^3 - x - 7$. Stop when your answer is accurate to 4 decimal places.

$f'(x) = 3x^2 - 1$

start $x_0 = 1$

$x_1 = 4.5$

$x_2 = 3.167\ldots$

$x_3 = 2.42\ldots$

$x_4 = 2.134\ldots$

x_5 2.087883

$x_6 = 2.086746$

$\rightarrow$ $x_7 = 2.086745$

$\vdots$

there is only one zero on this graph.