MTH 265, Final Exam, Summer 2024   Name _____

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

1. Find the volume of the region below $f(x, y) = \frac{1}{x^2 - y^2}$ for the region bounded by $x + y = 1$, $x + y = 2$, $x - y = 1$, $x - y = 4$. (18 points)

2. Write the integrals needed to find the center of mass for the region bounded by $0 \le z \le \frac{1}{1 + x^2 + y^2}$, $x^2 + y^2 \le 4$, $x \ge 0$, $y \ge 0$, $\rho = k \cos \theta$. Find the center of mass. (20 points)

3. Use the divergence theorem to evaluate $\int_S \int \vec{F} \cdot \vec{N} dS$ for $\vec{F}(x, y, z) = z^2\hat{\imath} + 2x^2\hat{\jmath} + 2xy\hat{k}$ for the closed surface bounded by $x^2 + y^2 = 1, z = 2$ and the coordinate planes. (20 points)

4. Use the second partials test to find any characterize any critical points for
$z = 4x + 6y - x^2 - y^2 + xy$. (12 points)

5. Find the absolute extrema of the function $f(x, y) = x^2 - 3xy + 2y^2 - 6y$ over the region bounded by $y = x^2, and \ y = \sqrt{x}$. (18 points)

6. Determine the maximum height and range of a projectile fired at height of 0.5 meters above the ground, with an initial velocity of 150 meters/second at an angle of $50°$ with the horizontal. Use the equation $\vec{r}(t) = (v_0 \cos\theta)t\hat{i} + \left[h_0 + (v_0 \sin\theta)t + \frac{1}{2}gt^2\right]\hat{j}$. (15 points)

7. Find the curvature of the curve $\vec{r}(t) = t^2\hat{i} + \cos t\,\hat{j} + \sin t\,\hat{k}$ at the point $t = \frac{\pi}{3}$. What is the radius of curvature at the same point? (14 points)

8. Use Green's Theorem to evaluate $\int_C (y - e^x)dx + (2x - \ln y)dy$ on the path described by the boundary of circle $x^2 + y^2 = 9$ oriented counterclockwise. (15 points)

9. Find the limit $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^3}$. (10 points)

10. Change the order of integration in $\int_0^4 \int_{x/4}^1 \frac{1}{1+y^4} dydx$ so that it can be integrated. Then complete the integration. (12 points)

11. Evaluate the line integral $\int_C y(x+z)ds$ on the path $\vec{r}(t) = \sqrt{t}\hat{\imath} + t^2\hat{\jmath} + 2t\hat{k}$ on $[1,4]$. (12 points)

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

12. Sketch the gradient field for $z = x^2 + y^2 + x^2 y + 4$ and use it to characterize any critical points. (18 points)

13. Write an equation of the ellipsoid $x^2 + \frac{y^2}{9} + z^2 = 1$ in parametric (surface) form. (10 points)

14. Find $\frac{dw}{dt}$ for $w = e^{xy} + yz, x = t^2, y = \frac{1}{t}, z = \ln t$ using the chain rule. Be sure your final answer contains only t. (12 points)

15. Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ for $x^3y + y^2 + \sqrt{x}\sin x - 8yz = z^2$. (15 points)

16. Find the position vector for $\vec{a}(t) = \frac{1}{t+1}\hat{\imath} - 4\hat{k}, \vec{v}(0) = \hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{r}(0) = 3\hat{\imath}$. (15 points)

17. Find the equation of the tangent plane to the surface $x^2z^2 = y^2$ at $(1,4,-2)$. (12 points)

18. Convert the triple integral $\int_{-6}^{6}\int_{0}^{\sqrt{36-x^2}}\int_{x^2+y^2}^{\sqrt{36-x^2-y^2}} \frac{\ln\sqrt{x^2+y^2+z^2+1}}{x^2+y^2+z^2}\, dzdydx$ to spherical coordinates and then complete the integration. Describe the region being integrated (over). (15 points)

19. Determine if the vector field $\vec{F}(x, y, z) = (\cos y + 3x^2)\hat{\imath} - (x \sin y + z)\hat{\jmath} - y\hat{k}$ is conservative. Find the potential function. (12 points)

Cylindrical

$$x = r \cos \theta$$
$$y = r \sin \theta$$
$$z = z$$
$$x^2 + y^2 = r^2$$
$$\tan^{-1}\left(\frac{y}{x}\right) = \theta$$

Spherical

$$x = \rho \cos \theta \sin \phi$$
$$y = \rho \sin \theta \sin \phi$$
$$z = \rho \cos \phi$$
$$x^2 + y^2 + z^2 = \rho^2$$
$$\tan^{-1}\left(\frac{y}{x}\right) = \theta$$
$$\cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right) = \phi$$
$$x^2 + y^2 = \rho^2 \sin^2 \phi = r^2$$

Dels

$$\frac{\partial}{\partial x} = \text{ partial derivative with respect to } x$$
$$\nabla f = grad\ f$$
$$\nabla^2 f = \nabla \cdot (\nabla f) = Laplacian\ of\ f$$
$$\nabla \cdot \vec{F} = div\ \vec{F}$$
$$\nabla \times \vec{F} = curl\ \vec{F}$$

Misc

$$ds = \|\vec{r}'(t)\| dt$$
$$\kappa = \frac{\|r' \times r''\|}{\|r'\|^3} = \frac{1}{R}$$
$$\vec{N} = \nabla F = \vec{r}_u \times \vec{r}_v$$
$$D = f_{xx}f_{yy} - \left(f_{xy}\right)^2$$