MTH 265, Quiz #11, Summer 2024 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the function $f(x, y) = x^2y - e^{x-y}$.
   a. Find the equation of the tangent plane at the point (1,1).

$F = x^2y - e^{x-y} - z$

$\nabla F = \langle 2xy - e^{x-y}, x^2 + e^{x-y}, -1 \rangle \quad \nabla F(1,1) = \langle 1, 2, -1 \rangle$

$f(x,y) \to f(1,1) = 1 - 1 = 0 \quad (1,1,0)$

$1(x-1) + 2(y-1) - 1(z-0) = 0$

   b. Find the equation of the normal line in vector-valued function form at the same point.

$\vec{n}(t) = (t+1)\hat{\imath} + (2t+1)\hat{\jmath} + (-t)\hat{k}$

2. Find the equation of the tangent plane for the parametric surface $\vec{r}(u, v) = u\cos v\,\hat{\imath} + (u\sin v - 1)\hat{\jmath} + u^2\hat{k}$ at $u = 2\sqrt{2}, v = \frac{\pi}{4}$. $(2,1,8)$

$\vec{r}_u = \cos v\,\hat{\imath} + \sin v\,\hat{\jmath} + 2u\hat{k}$

$\vec{r}_v = -u\sin v\,\hat{\imath} + u\cos v\,\hat{\jmath} + 0\hat{k}$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cos v & \sin v & 2u \\ -u\sin v & u\cos v & 0 \end{vmatrix} = (0 - 2u^2\cos v)\hat{\imath} - (0 + 2u^2\sin v)\hat{\jmath} + (u\cos^2 v + u\sin^2 v)\hat{k}$

$= -2u^2\cos v\,\hat{\imath} - 2u^2\sin v\,\hat{\jmath} + u\hat{k}$

$\langle -8\sqrt{2}, -8\sqrt{2}, 2\sqrt{2} \rangle$

$-8\sqrt{2}(x-2) - 8\sqrt{2}(y-1) + 2\sqrt{2}(z-8) = 0$

3. What kind of surface is the function in #2? Use technology to produce a graph.

paraboloid