MTH 265, Quiz #13, Summer 2024 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Use the Fundamental Theorem of Line Integrals to evaluate $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F}(x,y,z) = yze^{xz}\hat{\imath} + e^{xz}\hat{\jmath} + xye^{xz}\hat{k}$ on the curve $C: \vec{r}(t) = (t^2+1)\hat{\imath} + (t^2-1)\hat{\jmath} + (t^2-2)\hat{k}$, $0 \le t \le 2$.

$r(0) = \langle 1, -1, -2 \rangle$

$r(2) = \langle 5, 3, 2 \rangle$

$\int yze^{xz}\,dx = ye^{xz} + f(y,z)$

$\int e^{xz}\,dy = ye^{xz} + g(x,z)$

$\int xye^{xz}\,dz = ye^{xz} + h(x,y)$

$\varphi = ye^{xz}$

$\int_C F \cdot dr = \varphi(5,3,2) - \varphi(1,-1,2) = 3e^{10} - (-1)e^{-2} = 3e^{10} + \frac{1}{e^2}$

2. Use Green's Theorem to evaluate $\int_C xy^2dx + 2x^2ydy$ where C is the boundary of the region $y = x^2, y = x$.

M N

$\frac{\partial N}{\partial x} = 4xy \quad \frac{\partial M}{\partial y} = 2xy$

$\int_0^1 \int_{x^2}^{x} (4xy - 2xy)\,dy\,dx = \int_0^1 \int_{x^2}^{x} 2xy\,dy\,dx = \int_0^1 xy^2 \Big|_{x^2}^{x} dx$

$= \int_0^1 x^3 - x^5\,dx = \frac{1}{4}x^4 - \frac{1}{6}x^6 \Big|_0^1 = \frac{1}{12}$