MTH 265, Quiz #16, Summer 2024 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Use the chain rule to find $\frac{dz}{dt}$ for $z = \sqrt{1 + x^2 + y^2}, x = \ln t\,, y = \cos t$. Write your final answer in terms of t alone. You do not need to simplify.

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{1+x^2+y^2}} \qquad \frac{dy}{dt} = \frac{1}{t}$$

$$\frac{\partial z}{\partial y} = \frac{y}{\sqrt{1+x^2+y^2}} \qquad \frac{dy}{dt} = -\sin t$$

$$\frac{dz}{dt} = \frac{\ln t}{\sqrt{1+\ln^2 t+\cos^2 t}} \cdot \frac{1}{t} + \frac{\cos t}{\sqrt{1+\ln^2 t+\cos^2 t}}(-\sin t)$$

2. Use the chain rule to find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ for $z = e^r \cos\theta\,, r = st, \theta = \sqrt{s^2 + t^2}$. Write your final answers in term of t and s only. You do not need to simplify.

$$\frac{\partial z}{\partial r} = e^r \cos\theta \qquad \frac{\partial r}{\partial t} = s \qquad \frac{\partial \theta}{t} = \frac{t}{\sqrt{s^2+t^2}}$$

$$\frac{\partial z}{\partial \theta} = -e^r \sin\theta \qquad \frac{\partial r}{\partial s} = t \qquad \frac{\partial \theta}{s} = \frac{s}{\sqrt{s^2+t^2}}$$

$$\frac{\partial z}{\partial t} = e^{st}\cos\sqrt{s^2+t^2} \cdot s + -e^{st}\sin\sqrt{s^2+t^2} \cdot \frac{t}{\sqrt{s^2+t^2}}$$

$$\frac{\partial z}{\partial s} = e^{st}\cos\sqrt{s^2+t^2} \cdot t - e^{st}\sin\sqrt{s^2+t^2} \cdot \frac{s}{\sqrt{s^2+t^2}}$$

3. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $yz + x\ln y = z^2$ implicitly.

$$F = yz + x\ln y - z^2$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{-\ln y}{y-2z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{-z - x/y}{y-2z}$$