MTH 265, Quiz #5, Summer 2024 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the line integrals on the indicated paths.

   a. $\int_C xyz\,ds, C: x = 2\sin t, y = t, z = -2\cos t, 0 \le t \le 2\pi$ $\rightarrow \langle 2\cos t, 1, 2\sin t\rangle = r'$

   $ds = \sqrt{4\cos^2 t + 1 + 4\sin^2 t} = \sqrt{4+1} = \sqrt{5}\,dt$

   $\sqrt{5}\int_0^{2\pi} 4t \sin t \cos t\,dt = \sqrt{5}\int_0^{2\pi} 2t \sin 2t\,dt =$

   $\sqrt{5}\left[\frac{1}{2}\sin 2t - 2t\cos 2t\right]_0^{2\pi} = \sqrt{5}(-2\pi) = -2\sqrt{5}\pi$

   b. $\int_C \vec{F}\cdot d\vec{r}, \vec{F}(x,y) = xy\hat{\imath} - 3y^2\hat{\jmath}, \vec{r}(t) = 11t^2\hat{\imath} + t\hat{\jmath}, 0 \le t \le 1$

   $r'(t) = 22t\,\hat{\imath} + 1\hat{\jmath}$

   $F(t) = 11t^3\hat{\imath} - 3t^2\hat{\jmath}$

   $F\cdot dr = 242t^4 - 3t^2$

   $\int_0^1 242t^4 - 3t^2\,dt = \frac{242}{5}t^5 - \frac{3}{3}t^3\Big|_0^1 =$

   $\frac{242}{5} - 1 = 237/5$

2. Consider the function $f(x,y) = \ln(x + 2y^2) - \cos xy$. Find f_x, f_y, f_{xy}.

   $f_x = \frac{1}{x+2y^2}\cdot 1 + y\cdot\sin xy = \frac{1}{x+2y^2} + y\sin xy$

   $f_y = \frac{1}{x+2y^2}\cdot(4y) - x\sin xy = \frac{4y}{x+2y^2} + x\sin xy$

   $f_{xy} = (x+2y^2)^{-2}(-1)(4y) + \sin xy + xy\cos xy$

   $\frac{-4y}{(x+2y^2)^2} + \sin xy + xy\cos xy$

3. Calculate the total differential of $f(x,y,z) = xe^{yz}$, and then use the values of the function at $(1,4,0)$ to estimate the value of the function at the point $(0.95, 4.1, 0.01)$.

   $dx = -0.05$
   $dy = 0.1$
   $dz = 0.01$

   $dw = (e^{yz})dx + (xze^{yz})dy + (xye^{yz})dz$

   $= (e^0)(-0.05) + (0)\cancel{(0.1)} + 4e^0(0.01)$

   $= -0.05 + 0.04 = -0.01$

   $f(1,4,0) = 1e^0 = 1$

   $f(0.95, 4.1, 0.01) \approx$

   $1 - 0.01 = 0.99$