MTH 265, Quiz #6, Summer 2024 Name _____

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find ∇f and $\nabla^2 f$ for the function $f(x,y,z) = \frac{1}{2}xy^2\cos(y+z^3)$.

2. Find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ for $\vec{F}(x,y,z) = \sin(xy)\,\hat{\imath} - \cos(yz)\,\hat{\jmath} + \tan(xz)\hat{k}$.

3. Determine if the vector field $\vec{F}(x,y,z) = (x+y)\hat{\imath} + (y-z)\hat{\jmath} + z^2\hat{k}$ is conservative.

4. Consider the function $f(x,y) = \frac{x}{x^2+y^2}$. Sketch the following:
   a. The trace on the yz-plane.

b. The trace on the xz-plane.

c. 10 level curves.

d. Use technology to verify your level curves and produce a 3D graph of the function to verify your results. Attach the graphs to your submission.

5. Find the potential function, if it exists, for the vector field $\vec{F}(x, y, z) = (2xy + yz^2)\hat{\imath} + (x^2 - 2yz + xz^2)\hat{\jmath} + (2xyz - y^2 + \cos z)\hat{k}$. If not potential function exists, show work to prove that it is not.