MTH 265, Quiz #9, Summer 2024 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the integrals. Describe or sketch the volume defined by the integral.

   a. $\int_0^1\int_0^2\int_0^{\sqrt{1-z^2}}\frac{z}{y+1}\,dxdydz$

$$\int_0^1\int_0^2 \frac{z\sqrt{1-z^2}}{y+1}\,dy\,dz = \int_0^1 \ln|y+1|\, z\sqrt{1-z^2}\Big|_0^2 dz$$

$$(\ln 3-\cancel{\ln 1}^{\,0})\int_0^1 z\sqrt{1-z^2}\,dz = -\frac{1}{\cancel{2}}\ln 3\,(1-z^2)^{3/2}\cdot\frac{\cancel{2}}{3}\Big|_0^1 = +\frac{1}{3}\ln 3\,[0^{3/2}-1^{3/2}] =$$

$u=1-z^2$ $du=-2z$ $\int -\frac{1}{2}u^{1/2}du$

$$-\frac{1}{3}\ln 3$$

   b. $\int_0^{2\pi}\int_0^{\pi/2}\int_1^3 \rho^3\cos\phi\sin\phi\cos\theta\,d\rho d\phi d\theta$

$$\int_0^{2\pi}\int_0^{\pi/2}\frac{1}{4}\rho^4\Big|_1^3\cos\varphi\sin\varphi\cos\theta\,d\varphi\,d\theta = \frac{1}{4}(81-1)\frac{1}{2}\sin^2\varphi\Big|_0^{\pi/2}\int_0^{2\pi}\cos\theta\,d\theta$$

$$=10(1)\sin\theta\Big|_0^{2\pi} = 0$$

2. Change the integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\,dzdxdy$ to cylindrical coordinates and evaluate it. Sketch or describe the region of integration.

$x^2+y^2=4$

$z=\sqrt{x^2+y^2}=r$

$$\int_0^{2\pi}\int_0^2\int_r^2 r\cos\theta\, z\,dz\,r\,dr\,d\theta = \int_0^{2\pi}\int_0^2\frac{1}{2}z^2\Big|_r^2 r^2\cos\theta\,dr\,d\theta =$$

$$\int_0^{2\pi}\int_0^2\left(2-\frac{1}{2}r^2\right)r^2\cos\theta\,dr\,d\theta = \int_0^{2\pi}\int_0^2\left(2r^2-\frac{1}{2}r^4\right)\cos\theta\,dr\,d\theta =$$

$$\int_0^{2\pi}\left(\frac{2}{3}r^3-\frac{1}{10}r^5\Big|_0^2\right)\cos\theta\,d\theta = \int_0^{2\pi}\frac{32}{15}\cos\theta\,d\theta = \frac{32}{15}\sin\theta\Big|_0^{2\pi} = 0$$

3. Change the integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} xz\,dzdxdy$ to spherical coordinates and evaluate it. Sketch or describe the region of integration.

$\leftarrow \rho = 8$

$\rho = \pi/4$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sqrt{8}}\rho\cos\theta\sin\varphi\,\rho\cos\varphi\,\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta =$$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sqrt{8}}\rho^4\cos\theta\sin^2\varphi\cos\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{2\pi}\int_0^{\pi/4}\frac{1}{5}\rho^5\Big|_0^{\sqrt{8}}\cos\theta\sin^2\varphi\cos\varphi\,d\varphi\,d\theta$$

$$=\frac{64\sqrt{8}}{5}\int_0^{2\pi}\int_0^{\pi/4}\cos\theta\sin^2\varphi\cos\varphi\,d\varphi\,d\theta = \frac{128\sqrt{2}}{5}\cdot\frac{1}{3}\sin^3\varphi\Big|_0^{\pi/4}\cdot\int_0^{2\pi}\cos\theta\,d\theta =$$

$$\frac{128\sqrt{2}}{5}\cdot\frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)^3\sin\theta\Big|_0^{2\pi} = 0$$