

Instructions: Show all work. Use exact answers unless otherwise asked to round.

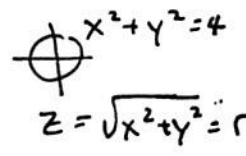
1. Evaluate the integrals. Describe or sketch the volume defined by the integral.

a. $\int_0^1 \int_0^2 \int_0^{\sqrt{1-z^2}} \frac{z}{y+1} dx dy dz$ $\int_0^1 \int_0^2 \frac{z\sqrt{1-z^2}}{y+1} dy dz = \int_0^1 \ln|y+1| z\sqrt{1-z^2} \Big|_0^2 dz$
 $(\ln 3 - \ln 1) \int_0^1 z\sqrt{1-z^2} dz = -\frac{1}{2} \ln 3 (1-z^2)^{3/2} \cdot \frac{2}{3} \Big|_0^1 = +\frac{1}{3} \ln 3 [0^{3/2} - 1^{3/2}] = -\frac{1}{3} \ln 3$
 $u = 1-z^2 \quad du = -2z \quad \int -\frac{1}{2} u^{3/2} du$

b. $\int_0^{2\pi} \int_0^{\pi/2} \int_1^3 \rho^3 \cos \phi \sin \phi \cos \theta \, d\rho d\phi d\theta$
 $\int_0^{2\pi} \int_0^{\pi/2} \frac{1}{4} \rho^4 \Big|_1^3 \cos \phi \sin \phi \cos \theta \, d\phi d\theta = \frac{1}{4} (81-1) \frac{1}{2} \sin^2 \phi \Big|_0^{\pi/2} \int_0^{2\pi} \cos \theta \, d\theta$
 $= 10(1) \sin \theta \Big|_0^{2\pi} = 0$

2. Change the integral $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 xz \, dz dx dy$ to cylindrical coordinates and evaluate it. Sketch or describe the region of integration.

$\int_0^{2\pi} \int_0^2 \int_r^2 r \cos \theta \, z dz r dr d\theta = \int_0^{2\pi} \int_0^2 \frac{1}{2} z^2 \Big|_r^2 r^2 \cos \theta \, dr d\theta = \int_0^{2\pi} \int_0^2 (2 - \frac{1}{2} r^2) r^2 \cos \theta \, dr d\theta = \int_0^{2\pi} \int_0^2 (2r^2 - \frac{1}{2} r^4) \cos \theta \, dr d\theta = \int_0^{2\pi} (\frac{2}{3} r^3 - \frac{1}{10} r^5) \Big|_0^2 \cos \theta \, d\theta = \int_0^{2\pi} \frac{32}{15} \cos \theta \, d\theta = \frac{32}{15} \sin \theta \Big|_0^{2\pi} = 0$



3. Change the integral $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} xz \, dz dx dy$ to spherical coordinates and evaluate it. Sketch or describe the region of integration.

$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{8}} \rho \cos \theta \sin \phi \, \rho \cos \phi \, \rho^2 \sin \phi \, d\rho d\phi d\theta = \int_0^{2\pi} \int_0^{\pi/4} \rho^4 \cos \theta \sin^2 \phi \cos \phi \, d\rho d\phi d\theta = \int_0^{2\pi} \int_0^{\pi/4} \frac{1}{5} \rho^5 \Big|_0^{\sqrt{8}} \cos \theta \sin^2 \phi \cos \phi \, d\phi d\theta = \frac{64\sqrt{8}}{5} \int_0^{2\pi} \int_0^{\pi/4} \cos \theta \sin^2 \phi \cos \phi \, d\phi d\theta = \frac{128\sqrt{2}}{5} \cdot \frac{1}{3} \sin^3 \phi \Big|_0^{\pi/4} \cdot \int_0^{2\pi} \cos \theta \, d\theta = \frac{128\sqrt{2}}{5} \cdot \frac{1}{3} \left(\frac{1}{\sqrt{2}}\right)^3 \sin \theta \Big|_0^{2\pi} = 0$

