MAT 277, Homework #3, Spring 2018 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Convert the equation in rectangular coordinates to both cylindrical and spherical coordinates. Simplify if possible. Solve for z in the cylindrical case (or z^2 as the case may be) or solve for ρ in the spherical case. Sketch a graph of the equation, or use technology to obtain the graph.
   a. $x^2+y^2+z^2=10$
   b. $x^2+y^2=9$
   c. $x^2+y^2-3z^2=0$
   d. $y=x^2$
   e. $x=4$

2. Convert the equation in cylindrical or spherical form to rectangular form. Simplify as much as possible and put into (some) standard form. Sketch a graph of the equation or use technology to obtain the graph.
   a. $r=2$
   b. $z=r^2\cos^2\theta$
   c. $\rho=2\sec\varphi$
   d. $\rho=2$
   e. $r=2\sin\theta$
   f. $r=\frac{1}{2}z$
   g. $\rho=4\csc\varphi\sec\theta$
   h. $\varphi=\frac{\pi}{6}$

3. Find a vector-valued function in two variables whose graph is the indicated surface. Use graphing software to create an equation of the graph.
   a. The plane $x+y+z=6$.
   b. The part of the plane $z=4$ that lines inside the cylinder $x^2+y^2=9$
   c. The ellipsoid $\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{1}=1$
   d. The cylinder $4x^2+y^2=16$
   e. The plane $z=y$

4. Find the rectangular form for the surface described below in vector-valued form. Sketch a graph of the equation or use technology to obtain the graph.
   a. $\vec{r}(u,v)=u\hat{\imath}+v\hat{\jmath}+\frac{v}{2}\hat{k}$
   b. $\vec{r}(u,v)=2ucos(v)\hat{\imath}+2usin(v)\hat{\jmath}+\frac{1}{2}u^2\hat{k}$
   c. $\vec{r}(u,v)=3\cos(v)\cos(u)\hat{\imath}+3\cos(v)\sin(u)\hat{\jmath}+5\sin(v)\hat{k}$
   d. $\vec{r}(u,v)=4\cos(u)\hat{\imath}+4\sin(u)\hat{\jmath}+v\hat{k}$
   e. $\vec{r}(u,v)=u\hat{\imath}+v\hat{\jmath}+\sqrt{uv}\hat{k}$

5. Graph the following parametric equations in 3D. For graphs that don't involve trigonometric functions, try $t=\{-3,-2,-1,0,1,2,3\}$. For points that contain trig functions you can try $t=\{-2\pi,-3\pi/2,-\pi,-\pi/2,0,\pi/2,\ \pi,3\pi/2,2\pi\}$ or if the graph is less predictable, try $t=\{0,\pi/4,\pi/2,3\pi/4,\pi,5\pi/4,3\pi/2,7\pi/4,2\pi\}$. Be sure to label each graph with an arrow on the curve in which t is increasing. Verify your sketch with online graphing tools.
   a. $\vec{r}(t)=t\hat{\imath}+(2t-4)\hat{\jmath}+(3t-7)\hat{k}$
   b. $\vec{r}(t)=\sin(t)\hat{\imath}+\cos(t)\hat{\jmath}+t\hat{k}$ (helix)
   c. $\vec{r}(t)=t^2\hat{\imath}+t\hat{\jmath}+4\hat{k}$
   d. $\vec{r}(t)=\sin^2(t)\,\hat{\imath}+t\hat{\jmath}+\cos(t^2)\,\hat{k}$

6. For each graph below, identify the equation of the function that produced it.

a. $\vec{r}(t) = t\cos t\,\hat{\imath} + t\hat{\jmath} + t\sin t\,\hat{k}$

b. $\vec{r}(t) = \cos^2 t\,\hat{\imath} + \sin^2 t\,\hat{\jmath} + t\hat{k}$

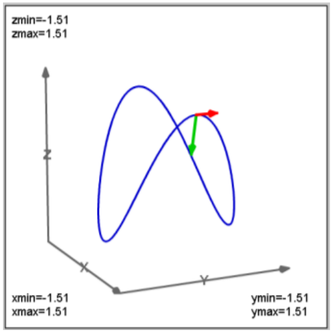
c. $\vec{r}(t) = t\hat{\imath} + e^t\hat{\jmath} + \cos t\,\hat{k}$

d. $\vec{r}(t) = t\hat{\imath} + \frac{1}{1+t^2}\hat{\jmath} + t^2\hat{k}$

e. $\vec{r}(t) = cos\,t\,\hat{\imath} + sin\,t\,\hat{\jmath} + \cos(2t)\,\hat{k}$

f. $\vec{r}(t) = 1\hat{\imath} + \cos t\hat{\jmath} + 2\sin t\,\hat{k}$

i.



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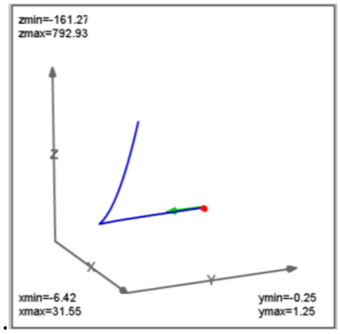

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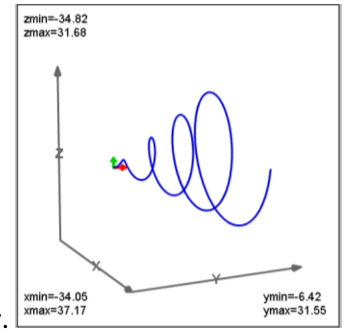

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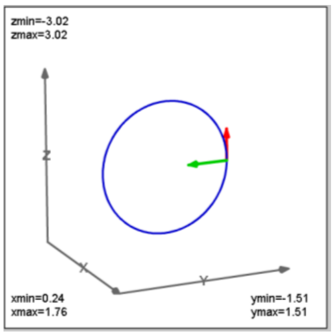

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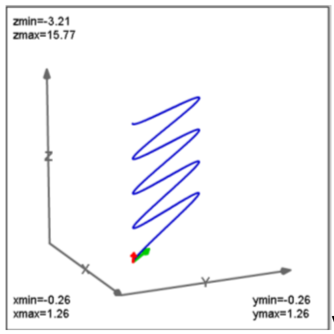

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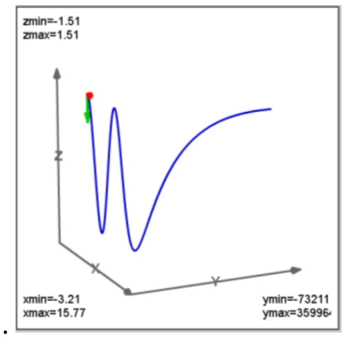


7. Find limits of each function at the indicated point. Be sure to check multiple paths. Use polar or spherical coordinates where appropriate.

a. $\displaystyle\lim_{(x,y)\to(2,1)} \frac{x-y-1}{\sqrt{x-y}-1}$

b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

c. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy}$

d. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4}$

e. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2\sin^2 x}{x^4+y^4}$

f. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^5}{2x^4+3y^{10}}$

g. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^3+2\sqrt{y}}{x^2+y^2}$

h. $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \arctan\left[\frac{1}{x^2+y^2+z^2}\right]$

i. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x-y}{\sqrt{x}-\sqrt{y}}$

j. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x-y^2}{2x^2+y}$

k. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^2}$

l. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$

m. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y\,e^y}{x^4+4y^2}$

n. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^3y^2}{x^6+y^4}$

o. $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2}$

p. $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2}$

8. Match the graphs below with their equations.

a. $z = x^2 - y^2$

b. $z = e^{-y}(x^2-1)$

e. $z = 10 - 4x - 5y$

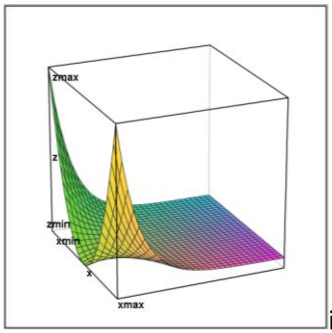
f. $z = (x^2-y^2)^2$

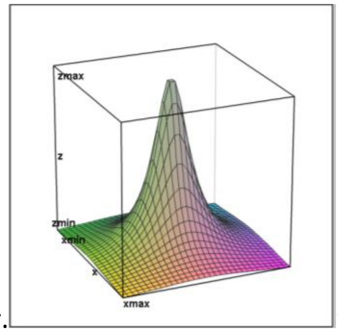
c. $z = \sin(x - y)\cos(x + y)$

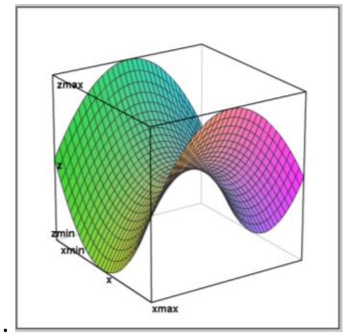
d. $z = \sin(|x| + |y|)$

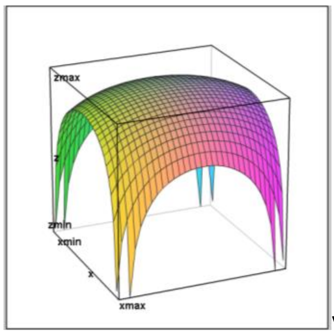
g. $z = \frac{4}{1+x^2+y^2}$

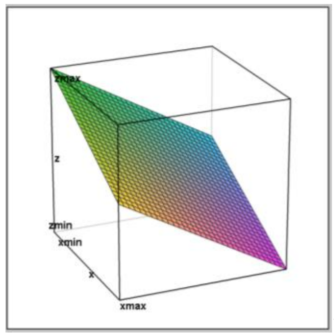
h. $z = \ln\left(9 - \frac{x^2}{10} - \frac{y^2}{20}\right)$

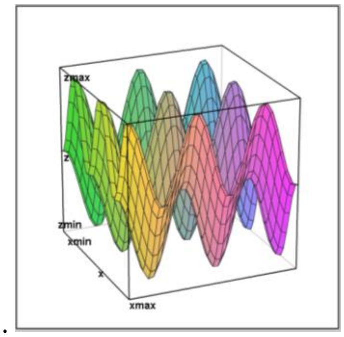
i. 

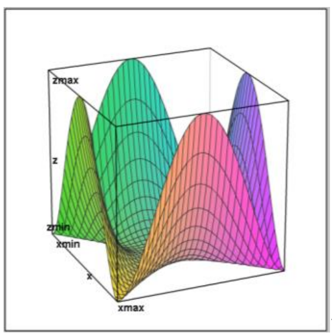
iv. 

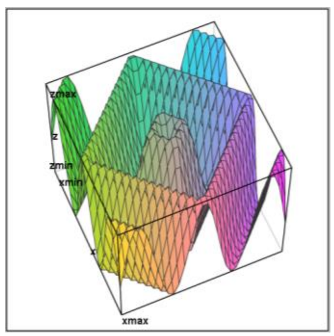
vii. 

ii. 

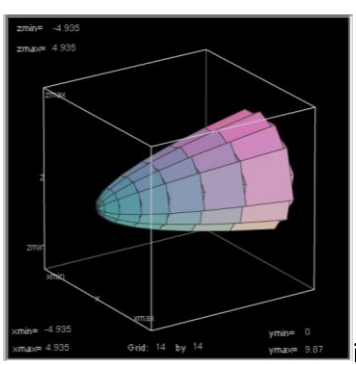
v. 

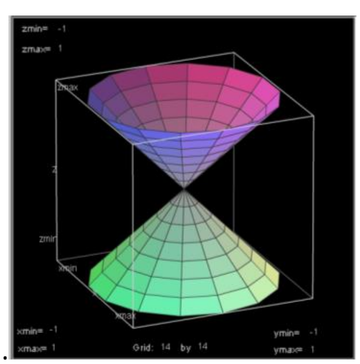
viii. 

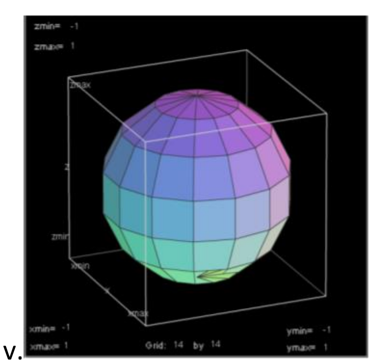
iii. 

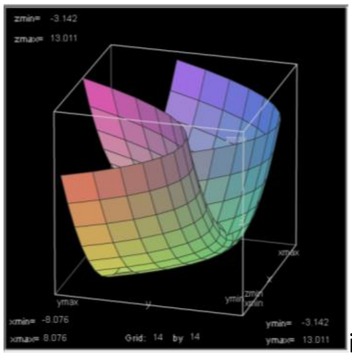
vi. 

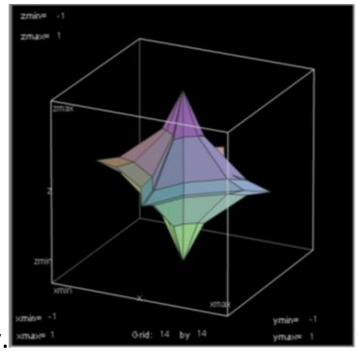
9. Match the graph of the parametric surface to its equation.
   a. $\vec{r}(u, v) = u\cos(v)\,\hat{\imath} + u\sin(v)\,\hat{\jmath} + u\hat{k}$
   b. $\vec{r}(u, v) = \sin(u)\cos(v)\,\hat{\imath} + \sin(u)\sin(v)\,\hat{\jmath} + \cos(u)\hat{k}$
   c. $\vec{r}(u, v) = u\sin(2v)\,\hat{\imath} + u^2\hat{\jmath} + u\cos(2v)\hat{k}$
   d. $\vec{r}(u, v) = \cos^3 u\cos^3 v\,\hat{\imath} + \sin^3 u\cos^3 v\,\hat{\jmath} + \sin^3 v\,\hat{k}$
   e. $\vec{r}(u, v) = (u + v)\hat{\imath} + (u^2 - v)\hat{\jmath} + (u + v^2)\hat{k}$
   f. $\vec{r}(u, v) = u\hat{\imath} + v\hat{\jmath} + (4 - u^2 - \sin(v))\hat{k}$

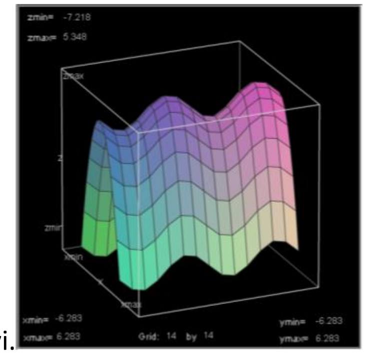
i. 

iii. 

v. 

ii. 

iv. 

vi. 

10. Match the graphs below to the equation that generated it in spherical coordinates.

a. $\rho = 4\sin(\phi)$

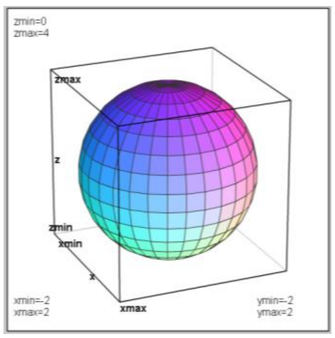
b. $\phi = \frac{\pi}{3}$

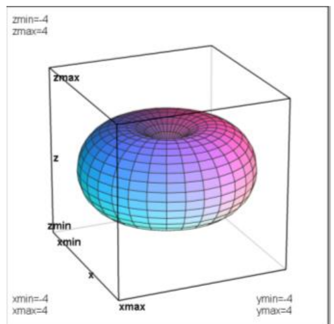
c. $\rho = \sin^2(\phi)\cos^2(\theta)$

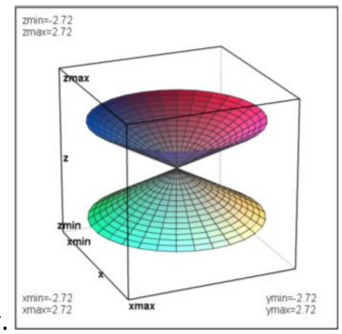
d. $\rho = 4\cos(\phi)$

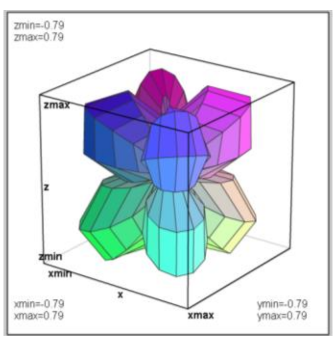
e. $\rho = \sin(\theta)\sin(\phi)$

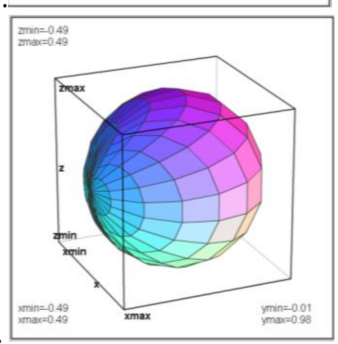
f. $\rho = \cos(\theta + \phi)\sin(\theta - \phi)$

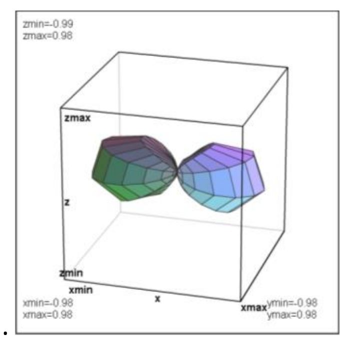
i. 

iii. 

v. 

ii. 

iv. 

vi. 

11. Determine the points at which the functions below are continuous.

a. $f(x,y,z) = \ln(x^2 + y^2 - 4)$

b. $h(x,y) = \begin{cases} \frac{xy}{x^2+xy+y^2}, & (x,y) \neq 0 \\ 0, & (x,y) = 0 \end{cases}$

c. $g(x,y,z) = \arcsin(x^2 + y^2 + z^2)$

d. $p(x,y) = \cos\left(\sqrt{1 + x - y}\right)$