MTH 277, Homework #1, Spring 2025 Name _____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the component vector given the magnitude and the angle the vector makes with the positive x-axis. Use the formula $r = \|\vec{v}\|, \vec{v} = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath}$. Round to two decimal places.
   a. $\left\|\vec{v}\right\| = 5, \theta = 120°$
   b. $\|\vec{v}\| = 8, \theta = -3.5\,rad$

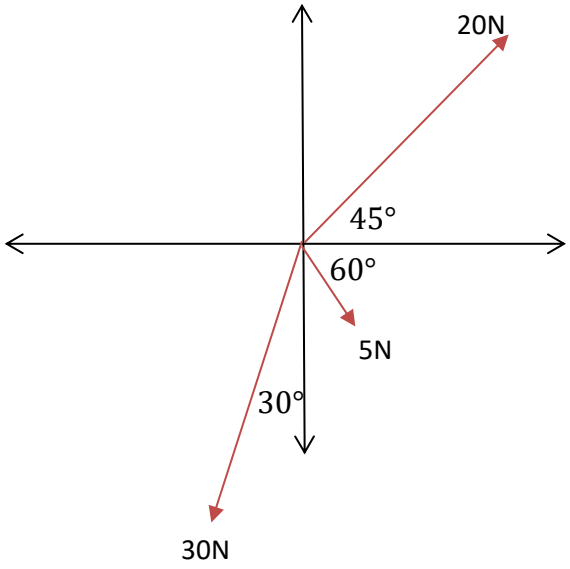
2. For the vector $20\hat{\imath} + 20\hat{\jmath}$, find the magnitude and direction of the vector.

3. Suppose that the given points (a, b, c) are vertices of a triangle in space: $A(5,3,4), B(7,1,3), C(3,5,3)$. Find the following.
   i. The vectors parallel to the sides: $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{BC}$
   ii. Find the lengths of the sides
   iii. Determine if the triangle is a right triangle, isosceles or neither.
   iv. Find the midpoint of the longest side.
   v. If the triangle is not a right triangle, say whether it is acute or obtuse. (Find the angles between the sides: $\overrightarrow{AB}$ *and* $\overrightarrow{AC}, \overrightarrow{BA}$ *and* $\overrightarrow{BC}, \overrightarrow{CA}$ *and* $\overrightarrow{CB}$; hint: note directionality of line segments!)
   vi. Find the area of the triangle.

4. Find the standard equation of the sphere; state the center and radius if not given.
   a. $x^2 + y^2 + z^2 + 6x - 2y + 10z = 19$
   b. $2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$
   c. Center at $(2, -4, 1)$, radius $\rho = 5$
   d. Endpoints of a diameter $(2,0,0)$ and $(0,6,0)$

5. Find the direction angles α, β, γ (the angle the vector makes with the axes) of the vector $\vec{u} = -4\vec{i} + 3\vec{j} + 5\vec{k}$.

6. Find the total force and direction resulting from adding the forces shown above.



7. Find the dot product and the cross product of the following pairs of vectors.
   a. $\vec{u} = \langle 1, 3, 4\rangle, \vec{v} = \langle 5, -2, -3\rangle$
   b. $\vec{u} = \langle 9, -4, 1\rangle, \vec{v} = \langle 3, -2, -5\rangle$

8. Find a unit vector orthogonal to $\langle -8, -6, 4\rangle$ and $\langle 10, -12, -2\rangle$.

9. Find the area of the parallelogram given by $(1,1,1), (2,3,4), (6,5,2), (7,7,5)$.

10. Find the volume of the parallelepiped whose sides are given by the vectors $\langle 2,1,0\rangle$, $\langle 0,2,1\rangle$, $\langle 0,-1,2\rangle$.

11. Find the projection of $\langle 1,-1,3\rangle$ onto the vector $\langle 3,4,-2\rangle$.

12. Find a vector orthogonal to $\langle 4,0,2\rangle$.

13. Find the work done by the force $\vec{F} = 8\hat{\imath} - 6\hat{\jmath} + 9\hat{k}$ that moves an object from the point $(0,10,8)$ to $(6,12,20)$ along a straight line if the distance is measured in meters and force in newtons.