MTH 277, Homework #10, Spring 2025 Name ____________________________________________

**Instructions**: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the velocity, speed, acceleration and jerk of a particle traveling along the path $\vec{r}(t)$. If a specific point is given, evaluate each derivative at the specified point.
   a. $\vec{r}(t)=t^2\vec{i}+t^3\vec{j},(1,1)$
   b. $\vec{r}(t)=3t\vec{i}+t\vec{j}+\frac{1}{4}t^2\vec{k}$
   c. $\vec{r}(t)=(t-\sin t)\vec{i}+(1-\cos t)\vec{j},(\pi,2)$
   d. $\vec{r}(t)=e^t\cos t\vec{i}+e^t\sin t\vec{j}+e^t\vec{k}$

2. Given the acceleration function, and initial conditions, find the velocity and position functions.
   a. $\vec{a}(t)=-\cos t\vec{i}-\sin t\vec{j}$, $\vec{v}(0)=\vec{j}+\vec{k},\vec{r}(0)=\vec{i}$
   b. $\vec{a}(t)=-32\hat{k},\vec{v}(0)=3\hat{\imath}-2\hat{\jmath}+\hat{k},\vec{r}(0)=5\hat{\jmath}+2\hat{k}$
   c. $\vec{a}(t)=e^t\hat{\imath}-8\hat{k},\vec{v}(0)=2\hat{\imath}+3\hat{\jmath}+\hat{k},\vec{r}(0)=\vec{0}$

3. Find the mass of the wire whose shape is given by $\vec{r}(t)$ and whose density is given by $\rho(x,y,z)$.
   a. $\vec{r}(t)=2\cos(t)\,\hat{\imath}+2\sin(t)\,\hat{\jmath}+t\hat{k},\rho(x,y,z)=|xy|,0\le t\le 4\pi$ [Hint: consider using symmetry to eliminate the absolute values.]
   b. $\vec{r}(t)=3\cos(t)\,\hat{\imath}+2t\hat{\jmath}+\sin(t)\,\hat{k},\rho(x,y,z)=kz,0\le t\le\pi$
   c. $\vec{r}(t)=(2t-4\sin(t))\hat{\imath}+(2-4\cos(t))\hat{\jmath},\rho(x,y,z)=\sin(t)\,,0\le t\le\pi$
   d. $\vec{r}(t)=t^2\hat{\imath}+2\sin^2(t)\,\hat{\jmath}+2\cot(t)\,\hat{k},\rho(x,y,z)=kyz,\frac{\pi}{4}\le t\le\frac{\pi}{2}$

4. Two functions are said to be *orthogonal* if $\int_{-1}^{1}\int_{-1}^{1}f(x,y)g(x,y)dydx=0$. (This isn't the only possible definition for orthogonal, but we will use it here.) Determine if the pair of functions are orthogonal.
   a. $f(x,y)=\sin(x)\cos(y)\,,g(x,y)=\cos(x)\sin(y)$
   b. $f(x,y)=e^y\sin(2x)\,,g(x,y)=e^y\sin(x)$
   c. $f(x,y)=\sinh(x)\,,g(x,y)=\cosh(y)$

5. Find the average value of the function $f(x,y)$ over the region R by finding $\bar{f}=\frac{1}{A}\int\int_R f(x,y)dA$.
   a. $f(x,y)=\sin(x+y)$ over the triangle with vertices (0,0), (0,1), (1,1).
   b. $f(x,y)=sin^2(x)$ over the region bounded by $y=x^2,y=4$.
   c. $f(x,y)=\cosh(x^2+y^2)$, over the cardioid $r=2+cos\theta$.

6. Find the velocity and position functions of a particle with $\vec{a}(t)=t^2\hat{\imath}+(\sin t-t\cos t)\hat{\jmath}+(\cos t+t\sin t)\hat{k},\vec{v}(0)=\hat{\imath},\vec{r}(0)=\hat{\jmath}-\hat{k}$. What is the speed of the particle at $t=\pi$?