MTH 277, Homework #11, Spring 2025 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the mass and center of mass of the lamina bounded by the graphs of the equations for the given density. Some of the integrals may be easier in polar coordinates.
   a. $x^2+y^2=a^2, x\geq 0, y\geq 0, \rho=k(x^2+y^2)$ c. $xy=4, x=1, x=4, y=0, \rho=kx^2$
   b. $y=\cos\frac{\pi x}{L}, y=0, x=0, x=\frac{L}{2}, \rho=k$ d. $r=1+\cos\theta, \rho=k$

2. Find the mass of the volume with the given density and bounded by the given surfaces.
   a. $z=4-x, x=0, x=4, y=0, y=4, z=0, \rho(x,y,z)=ky$
   b. $z=\frac{1}{y^2+1}, z=0, x=-2, x=2, y=0, y=1, \rho(x,y,z)=kz$

3. The value of the integral $I=\int_{-\infty}^{\infty} e^{-x^2/2}dx$ is required in the development of the normal probability density. A) use polar coordinates to evaluate the improper integral $I^2=\left(\int_{-\infty}^{\infty} e^{-x^2/2}dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2/2}dy\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}dA$. B) Use that information to determine *I*.

4. The average value of a function over a solid is given by $\frac{1}{V}\iiint_Q f(x,y,z)dV$ where V is the volume of the region Q. Find the average value for:
   a. $f(x,y,z)=z^2+4$ over the cube in the first octant bounded by the coordinate planes, and the planes $x=1, y=1$ and $z=1$.
   b. $f(x,y,z)=x+y$ over the solid bounded by the sphere $x^2+y^2+z^2=2$
   c. $f(x,y,z)=3x+2y-z+10$ over the solid bounded by the coordinate planes, $z=9-x^2, y=x$.

5. The joint density function for a pair of random variables is given by the functions below. Find the value of C in each case, and then use it to find the indicated probabilities.
   a. $f(x,y)=\begin{cases} Cx(1+y), & 0\leq x\leq 1,\ 0\leq y\leq 2 \\ 0, & otherwise \end{cases}$, $P\left(X\geq\frac{1}{2}\right), P\left(X\geq\frac{1}{2}, Y\leq\frac{1}{2}\right), P(X+Y\leq 1)$
   b. $f(x,y)=\begin{cases} Ce^{-\left(\frac{1}{2}x+\frac{1}{5}y\right)}, & x\geq 0,\ y\geq 0 \\ 0, & otherwise \end{cases}$, $P(Y\geq 1), P(X\leq 2, Y\leq 4)$