

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

- Convert the equation in rectangular coordinates to both cylindrical and spherical coordinates. Simplify if possible. Solve for z in the cylindrical case (or z^2 as the case may be) or solve for ρ in the spherical case. Sketch a graph of the equation, or use technology to obtain the graph.
 - $x^2 + y^2 + z^2 = 10$
 - $x^2 + y^2 = 9$
 - $x^2 + y^2 - 3z^2 = 0$
 - $y = x^2$
 - $x = 4$
- Convert the equation in cylindrical or spherical form to rectangular form. Simplify as much as possible and put into (some) standard form. Sketch a graph of the equation or use technology to obtain the graph.
 - $r = 2$
 - $z = r^2 \cos^2 \theta$
 - $\rho = 2 \sec \varphi$
 - $\rho = 2$
 - $r = 2 \sin \theta$
 - $r = \frac{1}{2} z$
 - $\rho = 4 \csc \varphi \sec \theta$
 - $\varphi = \frac{\pi}{6}$
- Find a vector-valued function in two variables whose graph is the indicated surface. Use graphing software to create an equation of the graph.
 - The plane $x + y + z = 6$.
 - The part of the plane $z = 4$ that lies inside the cylinder $x^2 + y^2 = 9$
 - The ellipsoid $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{1} = 1$
 - The cylinder $4x^2 + y^2 = 16$
 - The plane $z = y$
- Find the rectangular form for the surface described below in vector-valued form. Sketch a graph of the equation or use technology to obtain the graph.
 - $\vec{r}(u, v) = u\hat{i} + v\hat{j} + \frac{v}{2}\hat{k}$
 - $\vec{r}(u, v) = 2u\cos(v)\hat{i} + 2u\sin(v)\hat{j} + \frac{1}{2}u^2\hat{k}$
 - $\vec{r}(u, v) = 3\cos(v)\cos(u)\hat{i} + 3\cos(v)\sin(u)\hat{j} + 5\sin(v)\hat{k}$
 - $\vec{r}(u, v) = 4\cos(u)\hat{i} + 4\sin(u)\hat{j} + v\hat{k}$
 - $\vec{r}(u, v) = u\hat{i} + v\hat{j} + \sqrt{uv}\hat{k}$
- Graph the following parametric equations in 3D. For graphs that don't involve trigonometric functions, try $t = \{-3, -2, -1, 0, 1, 2, 3\}$. For points that contain trig functions you can try $t = \{-2\pi, -3\pi/2, -\pi, -\pi/2, 0, \pi/2, \pi, 3\pi/2, 2\pi\}$ or if the graph is less predictable, try $t = \{0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4, 2\pi\}$. Be sure to label each graph with an arrow on the curve in which t is increasing. Verify your sketch with online graphing tools.
 - $\vec{r}(t) = t\hat{i} + (2t - 4)\hat{j} + (3t - 7)\hat{k}$
 - $\vec{r}(t) = \sin(t)\hat{i} + \cos(t)\hat{j} + t\hat{k}$ (helix)
 - $\vec{r}(t) = t^2\hat{i} + t\hat{j} + 4\hat{k}$
 - $\vec{r}(t) = \sin^2(t)\hat{i} + t\hat{j} + \cos(t^2)\hat{k}$
- For each graph below, identify the equation of the function that produced it.

a. $\vec{r}(t) = t \cos t \hat{i} + t \hat{j} + t \sin t \hat{k}$

b. $\vec{r}(t) = \cos^2 t \hat{i} + \sin^2 t \hat{j} + t \hat{k}$

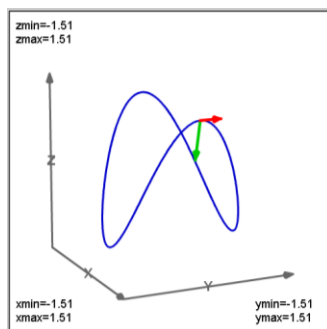
c. $\vec{r}(t) = t \hat{i} + e^t \hat{j} + \cos t \hat{k}$

d. $\vec{r}(t) = t \hat{i} + \frac{1}{1+t^2} \hat{j} + t^2 \hat{k}$

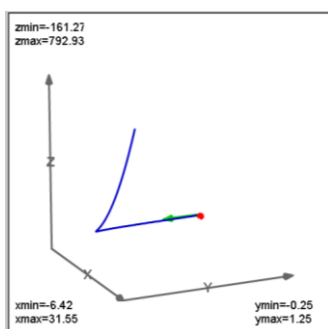
e. $\vec{r}(t) = \cos t \hat{i} + \sin t \hat{j} + \cos(2t) \hat{k}$

f. $\vec{r}(t) = 1 \hat{i} + \cos t \hat{j} + 2 \sin t \hat{k}$

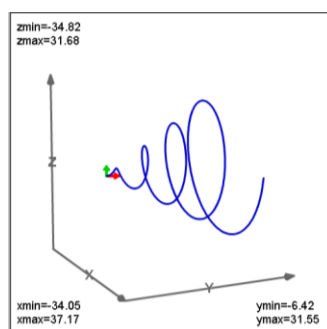
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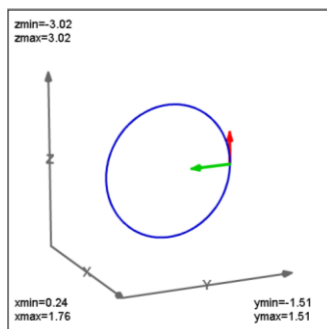
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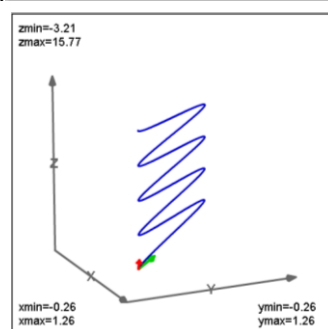
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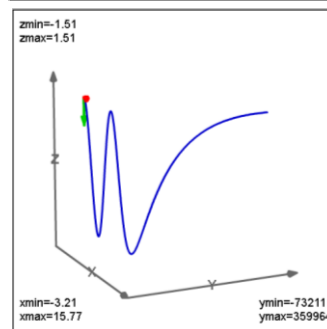
ii.



iv.



vi.



7. Find limits of each function at the indicated point. Be sure to check multiple paths. Use polar or spherical coordinates where appropriate.

a. $\lim_{(x,y) \rightarrow (2,1)} \frac{x-y-1}{\sqrt{x-y}-1}$

b. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$

c. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^2}{xy}$

d. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y^2}{x^4+3y^4}$

e. $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4+y^4}$

f. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y^5}{2x^4+3y^{10}}$

g. $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^3+2\sqrt{y}}{x^2+y^2}$

h. $\lim_{(x,y,z) \rightarrow (0,0,0)} \arctan \left[\frac{1}{x^2+y^2+z^2} \right]$

i. $\lim_{(x,y) \rightarrow (0,0)} \frac{x-y}{\sqrt{x}-\sqrt{y}}$

j. $\lim_{(x,y) \rightarrow (0,0)} \frac{2x-y^2}{2x^2+y}$

k. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^3+y^2}$

l. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3y}{x^6+y^2}$

m. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y e^y}{x^4+4y^2}$

n. $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^3y^2}{x^6+y^4}$

o. $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz}{x^2+y^2+z^2}$

p. $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2}$

8. Match the graphs below with their equations.

a. $z = x^2 - y^2$

b. $z = e^{-y}(x^2 - 1)$

e. $z = 10 - 4x - 5y$

f. $z = (x^2 - y^2)^2$

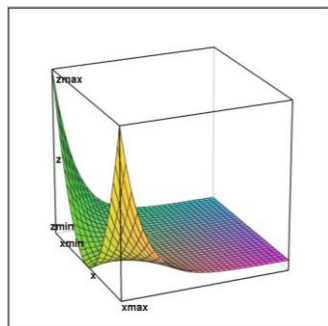
c. $z = \sin(x - y) \cos(x + y)$

g. $z = \frac{4}{1+x^2+y^2}$

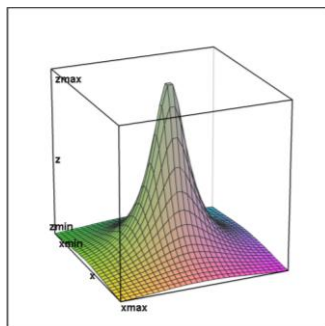
d. $z = \sin(|x| + |y|)$

h. $z = \ln\left(9 - \frac{x^2}{10} - \frac{y^2}{20}\right)$

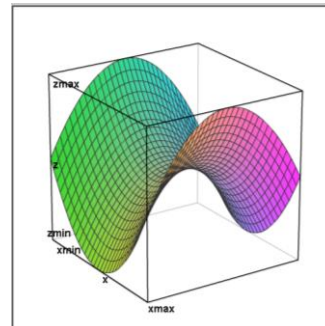
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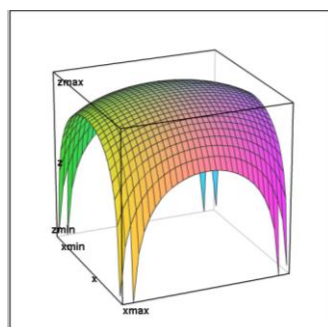
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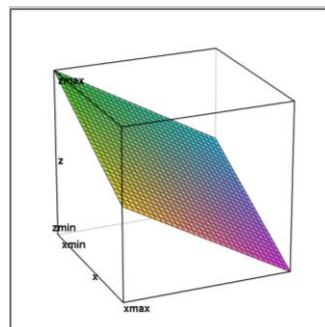
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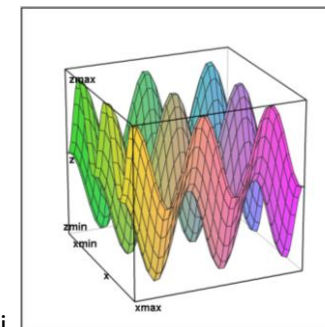
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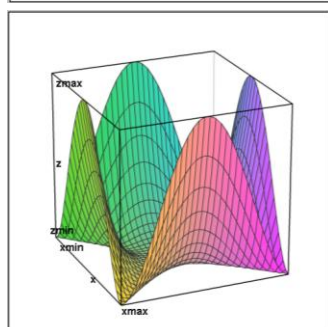
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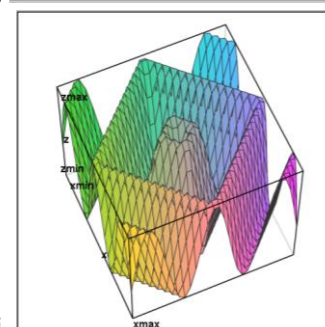
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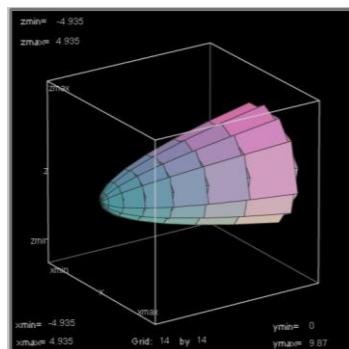
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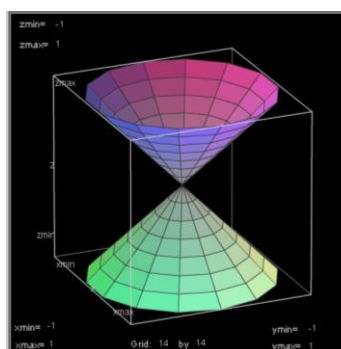
9. Match the graph of the parametric surface to its equation.

- $\vec{r}(u, v) = u \cos(v) \hat{i} + u \sin(v) \hat{j} + u \hat{k}$
- $\vec{r}(u, v) = \sin(u) \cos(v) \hat{i} + \sin(u) \sin(v) \hat{j} + \cos(u) \hat{k}$
- $\vec{r}(u, v) = u \sin(2v) \hat{i} + u^2 \hat{j} + u \cos(2v) \hat{k}$
- $\vec{r}(u, v) = \cos^3 u \cos^3 v \hat{i} + \sin^3 u \cos^3 v \hat{j} + \sin^3 v \hat{k}$
- $\vec{r}(u, v) = (u + v) \hat{i} + (u^2 - v) \hat{j} + (u + v^2) \hat{k}$
- $\vec{r}(u, v) = u \hat{i} + v \hat{j} + (4 - u^2 - \sin(v)) \hat{k}$

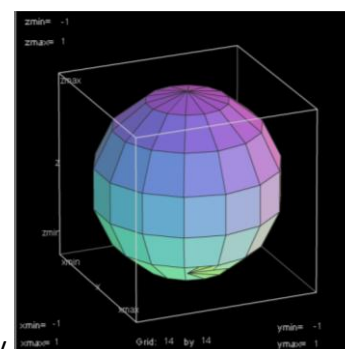
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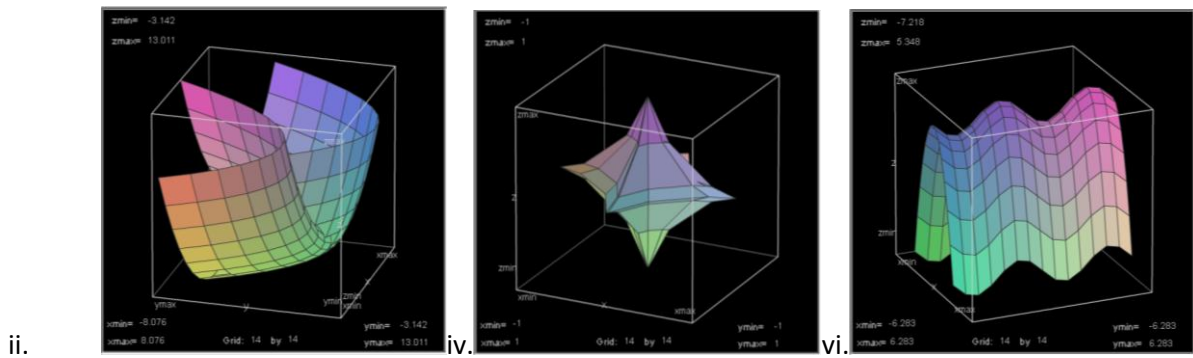


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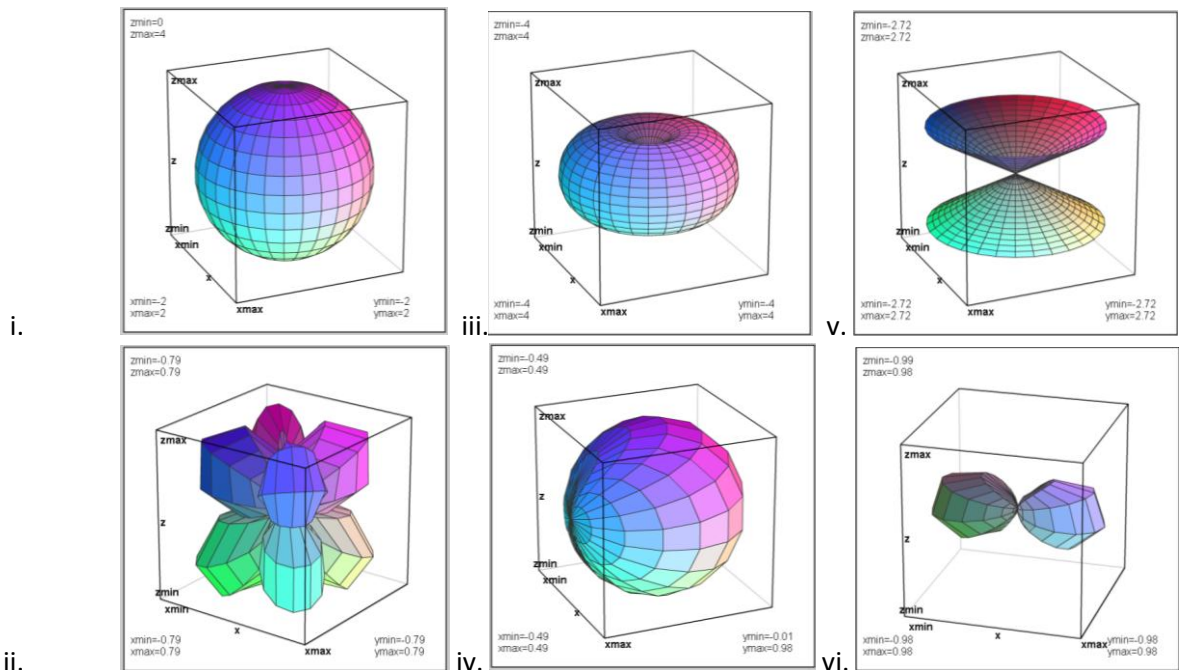
v.





10. Match the graphs below to the equation that generated it in spherical coordinates.

- a. $\rho = 4 \sin(\phi)$ d. $\rho = 4 \cos(\phi)$
 b. $\phi = \frac{\pi}{3}$ e. $\rho = \sin(\theta) \sin(\phi)$
 c. $\rho = \sin^2(\phi) \cos^2(\theta)$ f. $\rho = \cos(\theta + \phi) \sin(\theta - \phi)$



11. Determine the points at which the functions below are continuous.

- a. $f(x, y, z) = \ln(x^2 + y^2 - 4)$ c. $g(x, y, z) = \arcsin(x^2 + y^2 + z^2)$
 b. $h(x, y) = \begin{cases} \frac{xy}{x^2 + xy + y^2}, & (x, y) \neq 0 \\ 0, & (x, y) = 0 \end{cases}$ d. $p(x, y) = \cos(\sqrt{1 + x - y})$