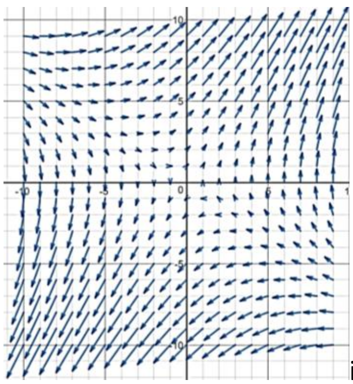
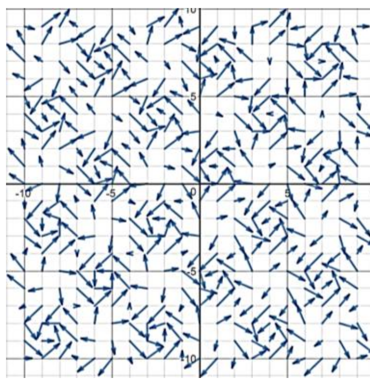
MAT 277, Homework #4, Spring 2025 Name ____

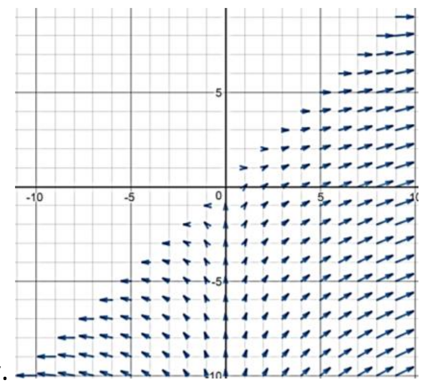
Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

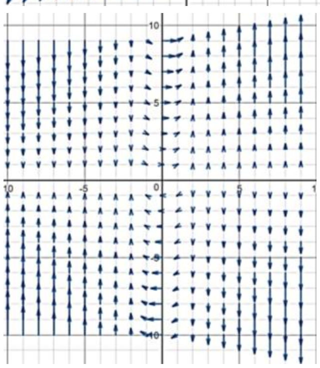
1. Plot the 2-dimensional vector fields below. Describe in words the shape of the field; what will happen to a particle placed in the field? Verify your direction field with technology.
   a. $\vec{F}(x,y)=x\vec{i}+y\vec{j}$
   b. $\vec{F}(x,y)=(x-1)\vec{i}-(y+2)\vec{j}$
   c. $\vec{F}(x,y)=x^2\vec{i}+xy\vec{j}$
   d. $\vec{F}(x,y)=(x^2+y^2)\vec{i}-xy\vec{j}$

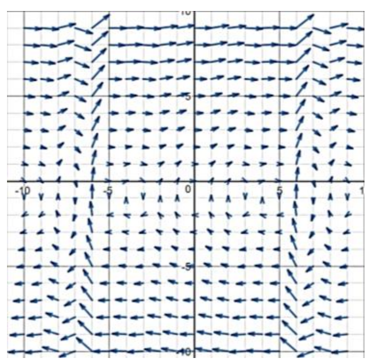
2. Match the vector field to the graph below.
   a. $\vec{F}(x,y)=y\hat{\imath}+(x+y)\hat{\jmath}$
   b. $\vec{F}(x,y)=y\hat{\imath}-x\hat{\jmath}$
   c. $\vec{F}(x,y)=(1+y)\hat{\imath}+\tan(x^2)\hat{\jmath}$
   d. $\vec{F}(x,y)=\cos(x+y)\hat{\imath}+\sin(x-y)\hat{\jmath}$
   e. $\vec{F}(x,y)=x\hat{\imath}+\sqrt{x-y}\hat{\jmath}$
   f. $\vec{F}(x,y)=ye^{-x^2}\hat{\imath}+\frac{xy}{5}\hat{\jmath}$

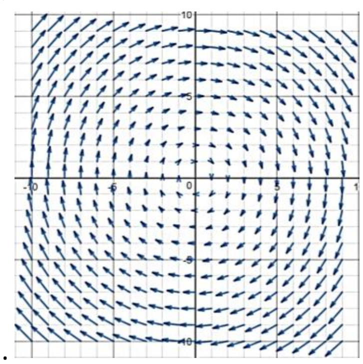
i. 

iii. 

v. 

ii. 

iv. 

vi. 

3. Use the line integral for area formula $A=\frac{1}{2}\int_C xdy-ydx$ to find the area for each of the closed curves.
   a. $x^2+y^2=9$ starting and ending at the point $(3,0)$ counterclockwise.
   b. Along the line segments connecting $(0,0)$, $(5,4)$, $(5,0)$ and back to $(0,0)$.
   c. Along the curve $y=x^2$ from $(0,0)$ to $(2,4)$, then along the line segment $(2,4)$ to $(0,4)$, and then back to the origin.

4. Evaluate each of the line integrals below on the indicated curve.
   a. $\int_C 3(x-y)ds, C:\vec{r}(t)=t\vec{i}+(2-t)\vec{j}, 0\le t\le 2$

b. $\int_C \rho(x,y,z)ds, \rho(x,y,z) = kz, \vec{r}(t) = t^2\vec{i} + 2t\vec{j} + t\vec{k}, 1 \le t \le 3$

c. $\int_C \vec{F} \cdot d\vec{r}, F(x,y) = x\vec{i} + y\vec{j}, C: \vec{r}(t) = t\vec{i} + t\vec{j}, 0 \le t \le 1$

d. $\int_C \vec{F} \cdot d\vec{r}, F(x,y,z) = yz\vec{i} + xz\vec{j} + xy\vec{k}, C: line\ \ from\ (0,0,0)\ to\ (5,3,2)$

e. $\int_C (x^2 + y^2)dx + 2xydy, \vec{r}_1(t) = t^3\vec{i} + t^2\vec{j}, 0 \le t \le 2; \vec{r}_2(t) = 2\cos t\vec{i} + 2\sin t\vec{j}, 0 \le t \le \frac{\pi}{2}$

5. For each vector-valued function, find $\vec{r}'(t), \vec{r}''(t), \int \vec{r}(t)dt, \int_0^a \vec{r}(t)dt$, $D_t\|\vec{r}'(t)\|, \int_1^2 \|\vec{r}'(t)\|dt$ for the specified value of a. When a is not specified, you should obtain a function with constants of integration. Use graphing software to obtain a graph of the function, and use the results you obtained for the derivatives to sketch the tangent and normal vectors on the graph.

a. $\vec{r}(t) = 6t\vec{i} - 7t^2\vec{j} + t^3\vec{k}$, $a = 1$

b. $\vec{r}(t) = (\sin t - t\cos t)\vec{i} + (\cos t + t\sin t)\vec{j} + t^2\vec{k}$, $a = \frac{\pi}{2}$

c. $\vec{r}(t) = e^t\vec{i} + \sec^2 t\vec{j} + \frac{1}{t^2+1}\vec{k}$, $a = 1$

d. $\vec{r}(t) = 4\sqrt{t}\vec{i} + t^2\sqrt{t}\vec{j} + \ln(t^2)\vec{k}$, $a = 4$

e. $\vec{r}(t) = \frac{1}{2}\vec{i} + \vec{j} - \vec{k}$

6. Find all first partial derivatives. Evaluate the derivatives at $(x,y) = (-1,1)$ or $(x,y,z) = (0,1,-2)$ as appropriate. Find any points where the first partials are all simultaneously zero (you may have to do this numerically or graphically).

a. $f(x,y) = x^2 - 3y^2 + 7$

b. $z = \frac{xy}{x^2 + y^2}$

c. $z = \sinh(2x + 3y)$

d. $f(x,y) = \arctan\left(\frac{y}{x}\right)$

e. $f(x,y,z) = \frac{3xz}{x+y}$

f. $f(x,y,z) = \frac{1}{\sqrt{1 - x^2 - y^2 - z^2}}$

g. $z = xe^{x/y}$

h. $z = e^y \sin xy$

i. $f(x,y) = \tanh(xy^2)$

j. $f(x,y) = x^2 + 4xy + y^2 - 4x + 16y + 3$

k. $w = 3x^2y - 5xyz + 10yz^2$

l. $z = \frac{e^x}{x+y^2}$

7. For parts a, c, j, k in the previous question, find all the second derivatives, f_{xy}, f_{yx} . Use this information to argue that the mixed partials are all equal. For part k, can the same be said of the mixed triples $f_{xyz}, f_{xzy}, f_{yxz}, f_{yzx}, f_{zxy}, f_{zyx}$?

8. Use the definition of the partial derivative to verify the first partial derivatives f_x, f_y of $f(x,y) = x^2 + 4xy + y^2 - 4x + 16y + 3$.

9. Show that the functions satisfy Laplace's Equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

a. $z = x^2 - y^2$

b. $z = e^y \sin(x)$

c. $z = \frac{y}{x^2+y^2}$

10. Find the partial derivatives of the parametric surfaces.

a. $\vec{r}(u,v) = u\cos v\,\hat{\imath} + u\sin v\,\hat{\jmath} + v\hat{k}$

b. $\vec{r}(u,v) = \sin v\,\hat{\imath} + \cos u \sin 2v\,\hat{\jmath} + \sin u \sin 2v\,\hat{k}$

c. $\vec{r}(u,v) = (1-u)(3+\cos v)\cos(4\pi u)\,\hat{\imath} + (1-u)(3+\cos v)(\sin 4\pi u)\,\hat{\jmath} + (3u + (1-u)\sin v)\hat{k}$

d. $\vec{r}(u,v) = (u^2+1)\hat{\imath} + (v^3+1)\hat{\jmath} + (u+v)\hat{k}$