MTH 277, Homework #6, Spring 2025   Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Evaluate the integral. Sketch or describe the region
   a. $\int_0^1\int_0^2 (x+y)dydx$
   b. $\int_0^2\int_{3y^2-6y}^{2y-y^2} 3ydxdy$
   c. $\int_0^3\int_0^\infty \frac{x^2}{1+y^2}dydx$
   d. $\int_0^1\int_1^{\ln(4)} \frac{\sinh(x)}{1+sinh^2(y)}dydx$
   e. $\int_0^{\pi/2}\int_0^{1-\cos\theta} \sin(\theta)rdrd\theta$
   f. $\int_1^4\int_1^{\sqrt{x}} 2ye^{-x}dydx$
   g. $\int_0^{\pi/4}\int_0^{\cos\theta} 3r^2\sin\theta drd\theta$
   h. $\int_0^4\int_0^{x/2} dydx+\int_4^6\int_0^{6-x} dydx$
   i. $\int_0^{\pi/2}\int_0^3 re^{-r^2}drd\theta$

2. Set up an integral and use it to evaluate the integral over the region R.
   a. $\iint_R \sin x\sin y dA$; R: rectangle with vertices $(-\pi,0),(\pi,0),(\pi,\pi/2),(-\pi,\pi/2)$
   b. $\iint_R xe^y dA$; R: triangle bounded by $y=4-x, y=0, x=0$
   c. $\iint_R (x^2+y^2)dA$; R: semicircle bounded by $y=\sqrt{4-x^2}, y=0$

3. Set up a double integral to find the volume of the solid bounded by the graphs of the equations.
   a. $z=xy, z=0, y=x, x=1$, first octant
   b. $z=\frac{1}{1+y^2}, x=0, x=2, y\geq 0$
   c. $x^2+y^2+z^2=r^2$

4. Use polar coordinates to find the volume of the solid bounded by the graphs of the equations.
   a. $z=xy, x^2+y^2=1, y\geq 0, x\geq 0, z\geq 0$
   b. $f(r,\theta)=1+\sin(r)$ over the region bounded by $r=2+\cos(\theta)$ between $0\leq\theta\leq\frac{\pi}{2}$.
   c. $z=\sqrt{16-2r^2}, inside\ the\ cylinder\ r=2\ and\ bounded\ by\ r=\sec(\theta)$ in the first octant.
   d. $z=\ln\left(x^2+y^2\right), z=0, x^2+y^2\geq 1, x^2+y^2\leq 4$

5. Evaluate the integral. Sketch or describe the region.
   a. $\int_{-1}^1\int_{-1}^1\int_{-1}^1 x^2y^2z^2dxdydz$
   g. $\int_1^4\int_1^{e^2}\int_0^{1/xz} \ln z dydzdx$

b. $\int_0^{\pi/2}\int_0^{y/2}\int_0^{1/y} \sin y\,dz\,dx\,dy$

c. $\int_0^{2}\int_{2x}^{4}\int_0^{\sqrt{y^2-4x^2}} dz\,dy\,dx$

d. $\int_0^{\pi/4}\int_0^{2}\int_0^{2-r} dz\,dr\,d\theta$

e. $\int_0^{\pi/2}\int_0^{\pi}\int_0^{\sin\theta} 2\cos\varphi\rho^2 d\rho\,d\theta\,d\varphi$

f. $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}}\sqrt{x^2+y^2}\,dz\,dy\,dx$ [Hint: you may want to change the coordinates for this integral.]

h. $\int_0^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^{x^2} y\,dz\,dy\,dx$

i. $\int_0^{3}\int_0^{x}\int_0^{9-x^2} dz\,dy\,dx$

j. $\int_0^{\pi/4}\int_0^{\pi/4}\int_0^{\cos\theta} \rho^2\sin\varphi\cos\varphi\,d\rho\,d\theta\,d\varphi$

k. $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^{3-r^2} r\,dz\,dr\,d\theta$

6. Change the order of integration of these integrals and integrate the result.
   a. $\int_0^1\int_x^1 x\sqrt{1+2y^3}\,dy\,dx$
   b. $\int_0^9\int_{\sqrt{x}}^3 \frac{4}{5+y^3}\,dy\,dx$
   c. $\int_0^1\int_{3x}^3 6e^{y^2}\,dy\,dx$

7. Set up an integral to find the volume of the solid.
   a. The solid that is bounded by the coordinate planes and $z = 9 - x^2, y = 2x$.
   b. $\int_0^{\pi/2}\int_0^{y/2}\int_0^{1/y} \sin y\,dz\,dx\,dy$

8. Find the volume of the solid using a triple integral and either cylindrical or spherical coordinates.
   a. The solid inside $x^2+y^2+z^2=16$ and outside $z=\sqrt{x^2+y^2}$.
   b. The solid that is the common interior below the sphere $x^2+y^2+z^2=80$ and above the paraboloid $z=\frac{1}{2}\left(x^2+y^2\right)$.
   c. The volume of the cone with height h and radius a.

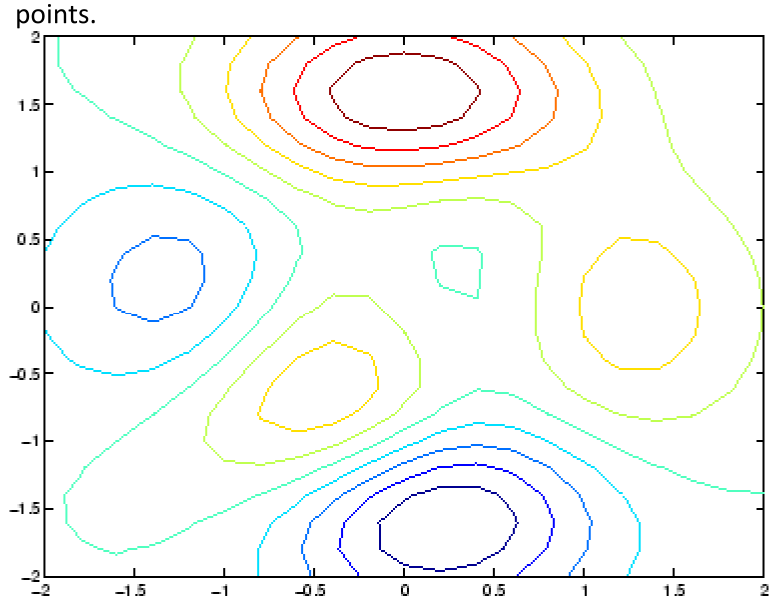
9. Change the order of integration and find the value of the integral.
   a. $\int_0^4\int_x^4 e^{-y^2}\,dy\,dx$
   b. $\int_0^1\int_{y^2}^1 \sqrt{x}\sin x\,dx\,dy$
   c. $\int_0^{\sqrt{\frac{\pi}{2}}}\int_y^{\sqrt{\frac{\pi}{2}}} \sin x^2\,dx\,dy$

10. Convert the integrals from rectangular to polar to integrate. Find the value of the integrals.
   a. $\int_0^1\int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}}(x^2+y^2)^{3/2}\,dy\,dx$
   b. $\int_0^6\int_0^{\sqrt{6y-y^2}} x^2\,dy\,dx$
   c. $\int_0^2\int_y^{\sqrt{8-y^2}} \sin\sqrt{x^2+y^2}\,dy\,dx$

11. Convert the following problems into the indicated coordinate system. Find the value of the integral.
    a. $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2}\, dzdydx$ into cylindrical
    b. $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2}\, dzdydx$ into spherical
    c. $\int_0^5 \int_0^{\sqrt{25-x^2}} \int_0^{\sqrt{25-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2}\, dzdydx$ into spherical
    d. $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{16-x^2-y^2}} \cos(x^2+y^2)\, dzdydx$ into cylindrical
    e. $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{16-x^2-y^2}} \cos\left(\sqrt{x^2+y^2+z^2}\right) dzdydx$ into spherical

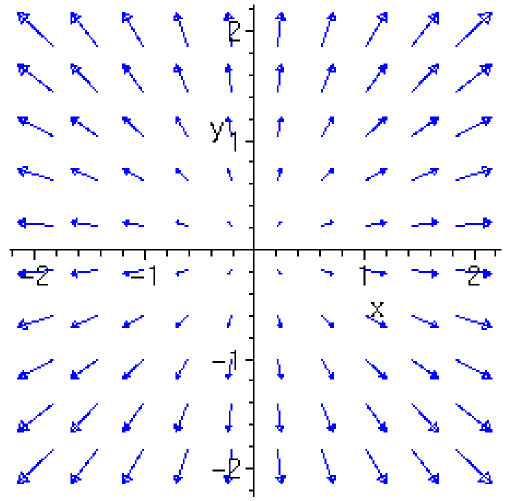
12. Find the area of one loop of $r = \cos 3\theta$ using a double integral.

13. Use the idea that gradients are perpendicular to level curves to sketch the gradient field for the graph below. Assume that blues are lower values, and reds are larger values to determine the direction of your vectors. Note any critical points or other interesting features, including saddle points.



14. For each of the gradient fields below, graph at least 5 level curves using the fact that the gradient is perpendicular to the level curves. Use that information to determine if the critical points are maxima, minima, or saddle points.

a.



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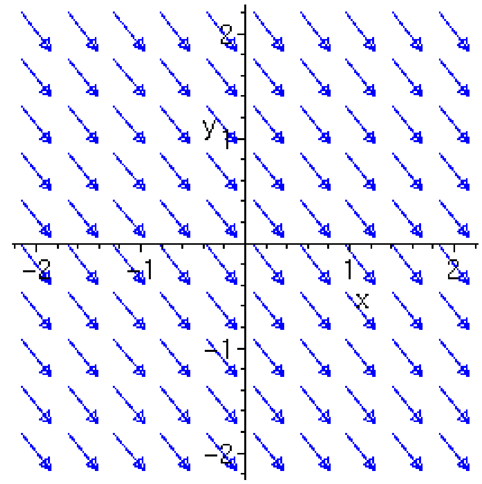
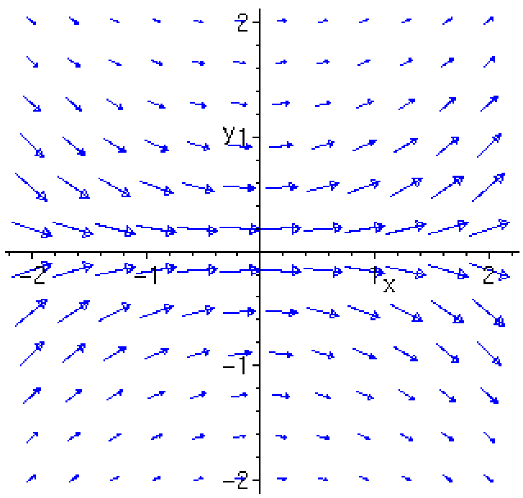


b.



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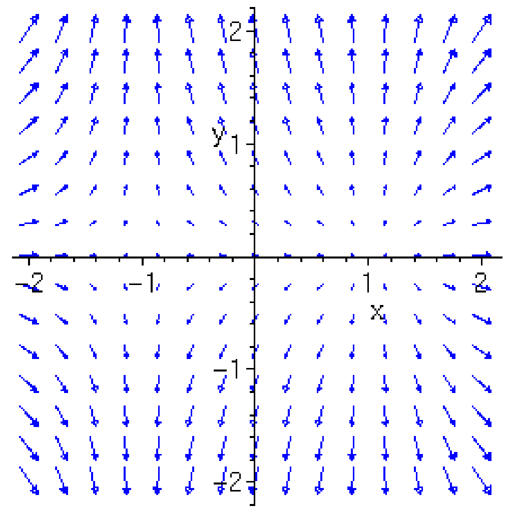


15. Following the method in the Gradients and Level Curves handout, for each of the following equations, analyze the graph by graphing the partial derivatives and analyzing the gradient field in each region of the plane. Label and categorize any critical points. Use technology to confirm your results.

a. $f(x,y) = xy$

b. $k(x,y) = 4 - x^2 - 4y^2$

c. $p(x,y) = \frac{x+y}{x^2+y}$

d. $r(x,y) = \frac{1}{x} + \frac{1}{y} + xy$

e. $t(x,y) = 4x^2 - y$

f. $g(x,y) = \sin(x)\cos(y)$

g. $m(x,y) = \frac{x}{1+x^2+y^2}$

h. $q(x,y) = 3x^3 - 12xy + y^3$

i. $s(x,y) = x^2 + 4xy + y^2 - 4x + 16y + 3$

16. Find and categorize any critical points using the second-partials test.

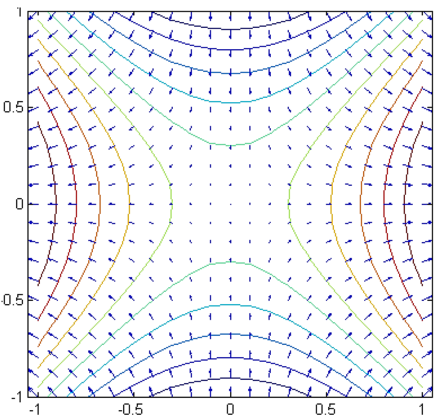
a. $f(x,y) = x^2 + xy + y^2 + y$

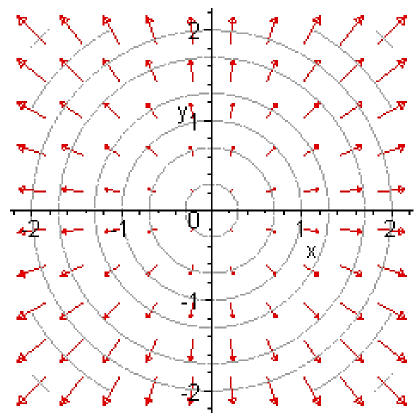
b. $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$

c. $f(x, y) = xy(1 - x - y)$
d. $f(x, y) = y^2 - 2y\cos(x)$

17. Find the absolute extrema for each function on the indicated region.
   a. $f(x, y) = x^2 + xy, R: \{(x, y) \mid -2 \le x \le 2, -1 \le y \le 1\}$
   b. $f(x, y) = 12 - 3x - 2y$ on the region bounded by the triangle with vertices (2,0), (0,1) and (1,2).
   c. $f(x, y) = 2x - 2xy + y^2, R: \{(x, y) \mid y \ge x^2, y \le 1\}$
   d. $f(x, y) = x^2 + y^2 + x^2y + 4, R: \{(x, y) \mid |x| \le 1, |y| \le 1\}$
   e. $f(x, y) = 2x^3 + y^4, R: \{(x, y) \mid x^2 + y^2 \le 1\}$

18. For each of the gradient fields below, characterize the critical point(s) as a maximum, minimum or saddle point.


a.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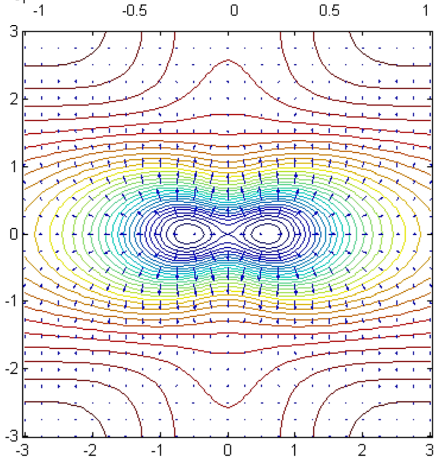
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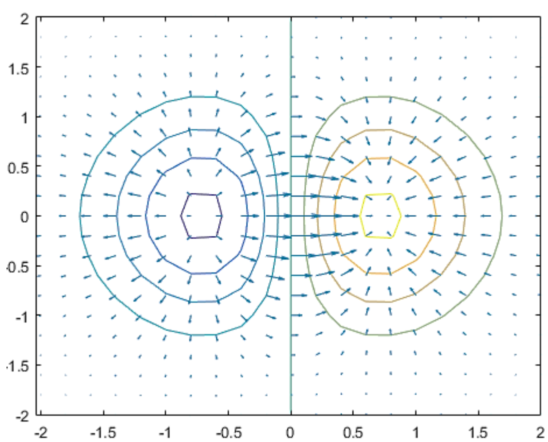
d.

19. Describe and sketch the level surfaces. Use technology to confirm your results.
   a. $f(x, y, z) = x + 3y + 5z$
   b. $f(x, y, z) = y^2 + z^2$
   c. $f(x, y, z) = x^2 + 3y^2 + 5z^2$
   d. $f(x, y, z) = x^2 - y^2 - z^2$