MTH 277, Homework #7, Spring 2025 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Find the length of the space curve over the given interval.
   a. $\vec{r}(t)=(t+1)\vec{i}+t^2\vec{j};[0,4]$
   b. $\vec{r}(t)=\vec{i}+t^2\vec{j}+t^3\vec{k};[0,2]$
   c. $\vec{r}(t)=a\cos t\vec{i}+a\sin t\vec{j};[0,2\pi]$
   d. $\vec{r}(t)=(\cos t+t\sin t)\vec{i}+(\sin t-t\cos t)\vec{j}+t^2\vec{k};\left[0,\frac{\pi}{2}\right]$
2. Find the curvature κ of the curve by two different methods. Which of the two methods is easier? Assume s is the arc length parameter. If a value of the parameter/variable is specified, evaluate the radius of curvature at that value. Also find the radius of curvature. Recall that $R\approx\frac{1}{\kappa}$.
   a. $\vec{r}(s)=(3+s)\vec{i}+\vec{j}$, $s=0$
   b. $\vec{r}(t)=4(\sin t-t\cos t)\vec{i}+4(\cos t+t\sin t)\vec{j}+\frac{2}{3}t^2\vec{k}$, $t=0$
   c. $\vec{r}(t)=5\cos t\vec{i}+4\cos t\vec{j}, t=\frac{\pi}{3}$
   d. $\vec{r}(s)=\left(1-\frac{1}{\sqrt{2}}s\right)\hat{\imath}+\left(1+\frac{1}{\sqrt{2}}s\right)\hat{\jmath}$, $s=1$
   e. $\vec{r}(t)=e^t\cos t\vec{i}+e^t\sin t\vec{j}+e^t\vec{k}$, $t=\pi$
3. Find the curvature κ for each function below. Sketch the curve and draw on the curve, at the indicated point, the circle that best approximates the curve at that point, using the radius of curvature.
   a. $\vec{r}(t)=t^2\hat{\imath}+\ln t\,\hat{\jmath}+t\ln t\,\hat{k}$, $(1,0,0)$
   b. $f(x)=xe^x, (1,e)$
4. Find $\frac{dw}{dt}$ for the following sets of equations using the chain rule. Be sure your final answers contain only t. Find $\frac{d^2w}{dt^2}$ for both.
   a. $w=xsin(y), x=e^t, y=\pi-t$
   b. $w=\cos(x-y), x=t^2, y=1$
5. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ for the following sets of equations using the chain rule. Be sure your final answer contains only t and s.
   a. $w=x^2+y^2, x=s+t, y=s-t$
   b. $w=xyz, x=s+t, y=s-t, z=st^2$
   c. $w=y^3-3x^2y, x=e^s, y=e^{t^2}$
6. Find the implicit derivative, or first partial derivatives of the following implicit functions. Calculate the derivatives the "long way" and using the formula. Verify that both produce the same result. Don't find more than you need. Simplify any complex fractions.
   a. $3x^2-xy+xz-2yz^3+z^4=\frac{z}{x}$
   b. $x^5-xyz+z^4=\ln\left(\frac{x}{y}\right)$
   c. $4x^3-\sin(xy)=ye^x$
   d. $2x^2w-xyz-2w\tan(zw)+e^{4w}=\frac{zy}{w}$