MTH 277, Homework #8, Spring 2025 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Use Green's Theorem to evaluate each line integral over a closed curve below.
    a. $\int_C 2xydx + (x+y)dy$ boundary of region between $y=0, y=1-x^2$
    b. $\int_C \cos y dx + (xy + x\sin y)dy$ boundary of region between $y=x, y=\sqrt{x}$

2. Find three positive numbers whose sum is 12, and the sum of whose squares is as small as possible.

3. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x+2y+3z=6$.

4. Find the shortest distance from the point $(2,0,-3)$ to the plane $x+y+z=1$.

5. Find the maximum and minimum volumes of a rectangular box whose surface area is $1500\ cm^2$ and whose total edge length is $200\ cm$.

6. Use the change of variables $x=2u+v, y=u+2v$ to evaluate the integral $\int\int_R (x-3y)dA$ over the triangular region given by the vertices $(0,0), (2,1), (1,2)$.

7. Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ for the indicated change of variables.
    a. $x=au+bv, y=cu+dv$
    b. $x=e^u \sin v, y=e^u \cos v$
    c. $u=x^2-y, v=y-x$
    d. $x=u\cos\theta - v\sin\theta, y=u\sin\theta + v\cos\theta$
    e. $x=\frac{u}{v}, y=u+v$

8. Find the Jacobian for the three-dimensional change of coordinates:

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

    a. $x=u(1-v), y=uv(1-w), z=uvw$
    b. $x=u-v+w, y=2uv, z=u+v+w$
    c. $x=rcos\theta, y=rsin\theta, z=z$

9. Set up a change of variables for the given regions. Sketch the regions before and after the transformation.
   a. Region bounded by $y = x^2 + 1, y = x^2 + 4, y = x, y = x + 4$
   b. Region bounded by parallelogram with vertices $(0,0), (2,2), (6,3), (4,1)$
   c. Region bounded by $xy = 1, xy = 4, y = 1, y = 4$
   d. Region bounded by $y = 2x, y = 3x, y = x^2, y = x^2 + 1$

10. For each function $f(x, y)$ calculate the region below it on the region described. Do this by changing to a convenient pair of variables. Sketch the region before and after the switch.
    a. $f(x, y) = y(x - y)$ on the parallelogram bounded by $(0,0), (3,3), (7,3), (4,0)$.
    b. $f(x, y) = e^{-\frac{xy}{2}}$ on the region bounded by $y = 2x, y = \frac{4}{x}, y = \frac{1}{4}x, y = \frac{1}{x}$.
    c. $f(x, y) = ysin(xy)$ on the region bounded by $xy = 1, y = 4, xy = 4, and\ y = 1$.

11. Use the method of Lagrange multipliers to maximize the function subject to the given constraint(s).
    a. $f(x,y) = x^2 - y^2; 2y - x^2 = 0$
    b. $f(x,y) = e^{xy}; y^2 + x^2 = 8$
    c. $f(x,y,z) = xyz; x + y + z - 6 = 0$
    d. $f(x,y) = xyz; x^2 + z^2 = 5, x - 2y = 0$