

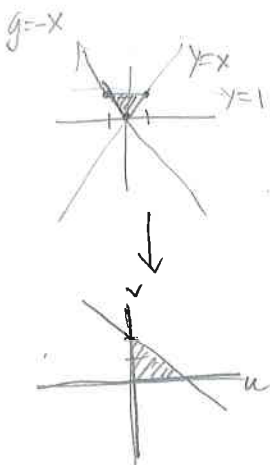
Name \_\_\_\_\_

KEY

Math 254, Quiz #14, Winter 2012

**Instructions:** Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Evaluate the integral  $\iint_R 4(x+y)e^{x-y} dA$  over the region bounded by the triangle whose vertices are  $(-1,1)$ ,  $(0,0)$  and  $(1,1)$ . Sketch the region before and after the change of variables. (10 points)



$$\begin{aligned} x+y=0 & \quad u=x+y & \quad \frac{1}{2}(u+v) = x \\ x-y=0 & \quad -v=-x+y & \quad \frac{1}{2}(u-v) = y \\ y=1 & \Rightarrow \frac{1}{2}(u-v) = 1 \\ & \quad u-v = 2 \\ & \quad v = u+2 \end{aligned}$$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

$$\int_0^2 \int_0^{u+2} 4ue^v \left(-\frac{1}{2}\right) dv du$$

$$-2 \int_0^2 ue^v \Big|_0^{u+2} du =$$

$$-2 \int_0^2 ue^{u+2} - u du$$

$$\begin{aligned} p &= u & dq &= e^{u+2} \\ dp &= du & q &= e^{u+2} \end{aligned}$$

$$-2 \left[ ue^{u+2} - \int_0^2 e^{u+2} du - \int_0^2 u du \right] =$$

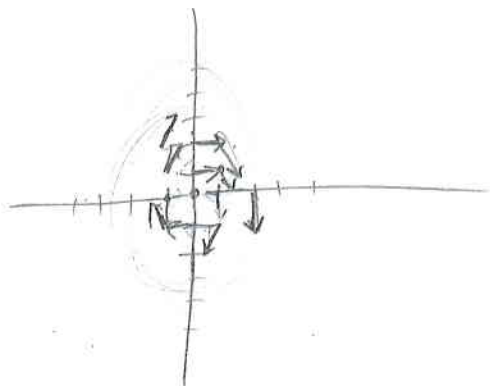
$$-2 \left[ ue^{u+2} - e^{u+2} - u \right]_0^2 =$$

$$-2 \left[ 2e^4 - e^4 - 2 - 0 + e^0 - 0 \right] =$$

$$-2 \left[ 2e^4 - e^4 - 2 + 1 \right] = \boxed{-2[e^4 - 1]}$$

2. Sketch the graph of the vector field  $\vec{F}(x,y) = y\hat{i} - 2x\hat{j}$  (using at least 15 points, use the back of the page). Determine if the vector field is conservative. (5 points)

x	y	F
0	0	$\langle 0, 0 \rangle$
1	0	$\langle 0, -2 \rangle$
0	1	$\langle 1, 0 \rangle$
-1	0	$\langle 0, 2 \rangle$
0	-1	$\langle -1, 0 \rangle$
1	1	$\langle 1, -2 \rangle$
1	-1	$\langle -1, 2 \rangle$
-1	-1	$\langle -1, 2 \rangle$
-1	1	$\langle 1, 2 \rangle$
1	2	$\langle 2, -2 \rangle$
2	0	$\langle 0, -4 \rangle$
-1	2	$\langle 2, 2 \rangle$
0	2	$\langle 2, 0 \rangle$



$$\frac{\partial}{\partial y} [y] = 1$$

$$\frac{\partial}{\partial x} [-2x] = -2$$

$$1 \neq -2$$

not conservative