Name KEY

Math 254, Quiz #16, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Find the curl and divergence of the vector field $\vec{F}(x,y,z) = 3x^2y^2\hat{i} + 2x^3y\hat{j} + \hat{k}$. (5 points)

Curl $= \vec{0}$ (see quiz 15)

$\nabla \cdot F = 6xy^2 + 2x^3$

2. Use Green's Theorem to evaluate the line integral $\int_C \overset{M}{(x-3y)}dx + \overset{N}{(x+y)}dy$ along the boundary of the region between the circles $x^2+y^2=1, x^2+y^2=4$. (5 points)

$\frac{\partial N}{\partial x} = 1 \qquad \frac{\partial M}{\partial y} = -3$

$1-(-3) = 4$

$\int_0^{2\pi}\int_1^2 4\, r\,dr\,d\theta = 4(\pi 4 - \pi) = \boxed{12\pi}$

3. Find the surface area of the graph described parametrically by $\vec{r}(u,v) = u\hat{i} + \frac{1}{4}v^3\hat{j} + uv\hat{k}$ for $0 \le u \le 2, 0 \le v \le 2$. If the integral you obtain can't be evaluated easily, you may do it numerically. (5 points)

$r_u = \hat{i} + v\hat{k}$

$r_v = \frac{3}{4}v^2\hat{j} + u\hat{k}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & v \\ 0 & \frac{3}{4}v^2 & u \end{vmatrix} = (0 - \frac{3}{4}v^3)\hat{i} - (u-0)\hat{j} + (\frac{3}{4}v^2 - 0)\hat{k}$

$\sqrt{\frac{9}{16}v^6 + u^2 + \frac{9}{16}v^4}$

$\int_0^2\int_0^2 \sqrt{\frac{9}{16}v^6 + u^2 + \frac{9}{16}v^4}\, du\, dv = 9.32808$