RECONCEPTUALIZING MULTIVARIABLE CALCULUS

EMPHASIZING CONCEPTUAL CONTINUITY

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AN EXPERIMENT

LET ME TELL YOU ABOUT AN EXPERIMENT I'VE BEEN CONDUCTING…

BACKGROUND

- For students going on to Physics or Engineering, Multivariable Calculus is perhaps one of their most important courses.
- Electricity and Magnetism employs advanced topics like surface integrals
- E&M and Quantum Mechanics employs PDEs
- Engineering Statistics employs multivariate functions and integrals
- Even Classical Mechanics employs tangent vectors and plane approximations

- Students often, however, leave the course struggling to use concepts from multivariable calculus effectively in these courses.

BACKGROUND

- Trouble appears before students even leave the course
  - Students do the hardest material in the course last with insufficient time to master it before forgetting it
  - Students learn related material (e.g. tangent planes) in different chapters and at different times and at final exam time are unable to tell the different situations apart, having never been asked to do that before
  - Addressing these issues in ad hoc ways did not appear to be effective

EXPERIMENT

- In an attempt to address this problem, I have
  - Arranged material in my multivariable calculus courses in a new way
  - The new arrangement emphasizes related concepts rather than computational modes
  - It also moves forward vector analysis topics like surface integrals to give students the necessary time to visit and revisit them prior to leaving the course

TRADITIONAL SEQUENCE OF TOPICS

- Precisely where and how multivariable calculus begins varies a bit by course and institution and textbook, but in general, the follow sequence of topics are addressed:
  - Vectors (operations on vectors, dot product, cross product, 3D vectors, etc.)
  - Vector-Valued Functions (derivatives, integrals, tangent/normal vectors, curvature, etc.)
  - Partial Derivatives (gradient, directional derivative, extrema, limits, chain rule, etc.)
  - Multiple Integrals (change of variables, volume, center of mass, surface area, etc.)
  - Vector Analysis (line integrals, surface integrals, parametric surfaces, etc.)
- This sequence of topics is basically universal in at least a dozen calculus texts I’ve examined.

TRADITIONAL SEQUENCE OF TOPICS

- In each of the main chapter topics (Vector-Valued Functions, Partial Derivatives, Multiple Integrals, and Vector Analysis) the chapters follow the same general sequence of material in each
  - Introduce any algebra needed for the material to come
  - Introduce the basic computational tool(s) employed in the chapter
  - Develop the theoretical extensions of that tool
  - Apply it to problems or other applications
- In some cases theory/applications are interchanged, but this holds in general.

TRADITIONAL SEQUENCE OF TOPICS

- Consider the topics for the chapter on Partial Derivatives from Larson & Edwards, Calculus: Early Transcendental Functions, 5th ed.

Chapter 13: Functions of Several Variables

13.1: Introduction to Functions of Several Variables (31)

13.2: Limits and Continuity (39)

13.3: Partial Derivatives (43)

13.4: Differentials (31)

13.5: Chain Rules for Functions of Several Variables (31)

13.6: Directional Derivatives and Gradients (36)

13.7: Tangent Planes and Normal Lines (33)

13.8: Extrema of Functions of Two Variables (37)

13.9: Applications of Extrema of Functions of Two Variables (35)

13.10: Lagrange Multipliers (32)

TRADITIONAL SEQUENCE OF TOPICS

- Compared to a course like Calculus I (Differential Calculus), Multivariable Calculus is typically laid out like four mini-courses, each one following the general sequence of 1) algebra you need now, 2) computational skills, 3) develop theory, 4) application.

MY EXPERIMENT

WHAT IF MULTIVARIABLE CALCULUS WAS TREATED LIKE A SINGLE UNIFIED TOPIC?

MOTIVATION FOR RE-SEQUENCING

- Treat Multivariable Calculus as a single "thing" instead of 4 smaller topics
- Approach entire course in 4 stages:
  - Algebra
  - Basic computational skills
  - Theory
  - Applications
- Give students a chance to compare and contrast immediately conceptually similar the same day or the same week.
- Never be very far from any particular computational method and so harder to forget.
- Cycling through methods multiple times would reinforce skills rather than weaken them.

METHODS

- Before undergoing a radical redesign, alternative methods were first attempted to address issues. While some success was achieved in this process, not enough in my estimation.
- Consultation with colleagues who frequently teach the course was undertaken prior to implementation. (Much thanks for both Dr. Gary Gutman and Ken Seidel for their feedback.)
- About a year deciding if I was really crazy enough to try this.
- Investigation of best learning strategies to estimate impact of changes.

NEW ORDER OF TOPICS

Following the

- 1) Algebra,
- 2) Basics,
- 3) Theory,
- 4) Application

scheme, let’s look at each section in detail.

ALGEBRA

- The algebra section covers about 20% of the course and collects algebraic topics together that will be needed for the entire course.
  - Review of Vectors including length, dot and cross products
  - Optional introduction to matrices (primarily matrix vector products and determinants)
  - Plotting points and curves and in three dimensions, parametric and vector-valued functions
  - Introduction to functions of more than one variable, standard 3D surfaces, trace and level curves
  - Cylindrical and spherical coordinates; parametric surfaces
  - Graphing vector fields
  - Limits and continuity

BASIC COMPUTATIONAL SKILLS

- This section covers all the basics and definitions of calculus of functions of several variables/multiple dimensions. It covers about 30% of the course.
  - Derivatives and integrals on vector-valued functions
  - Introduction to line integrals
  - Partial derivatives
  - Gradients, Curl, Divergence
  - First and Second Derivative Tests (gradient fields and 2^{nd} partials test)
  - Potential functions
  - Iterated integrals (rectangular, polar, cylindrical and spherical coordinates, changing limits, volume)

THEORETICAL DEVELOPMENT

- This section develops the theoretical machinery and begins addressing elementary applications, including the vector analysis. This section covers about 40% of the course.
  - Arc Length and Surface Area
  - Tangent and Normal Vectors (vector-valued, 2-or-more variables, parametric surfaces)
  - Differentials and Linear approximations
  - Curvature
  - Green's, Divergence, Stokes' Theorems
  - Implicit Differentiation and Chain Rule
  - Change of Variables

APPLICATIONS

- This section covers applications that span the course, for the remaining 10% or so.
  - Projectile Motion
  - Centers of Mass
  - Lagrange Multipliers
  - Applications to Statistics (averages, etc.)

INITIAL RESULTS

- I taught the course for the first time this way in Spring 2014.
- While the class was small, student performance on problem areas appeared to improve at first exposure and at the final.
- Exam performance suggested computational skills were strengthened by the need to constantly revisit them throughout the course.
- Student evaluations were extremely positive.
- Students taking engineering statistics along with my course reached multiple integrals prior to requiring them in statistics. The traditional sequence aligned less advantageously.

STUDENT QUOTES

- “Handouts given in class were helpful in understanding the material as well as Betsy’s restructuring of the textbook material. The order that she used helped in utilizing certain math mechanics and applications in a logical sequence to better understand the material.”
- “I like that the professor covered the book’s chapters in a different order so that the material made more sense.”
- “The reorganization of the chapters seemed much more sensible than the order of chapters in the book.”
- Comment on the elements of the course you liked: “The structure of the class (what sections were taught when, etc.).”

POTENTIAL DRAWBACKS

- While kinks are still being worked out, some drawbacks to this approach exists.
  - As far as I know, no textbook or resource employs this strategy so bouncing around a textbook is necessary. Often this requires doing only part of a section early on, and then returning to the rest of it at a later time, or sometimes more than once to complete it. Some people (faculty and students) have a great deal of difficulty with the apparent non-linearity.
  - Small differences across textbooks make switching texts considerably more difficult.

GOING FORWARD

- Many sections I work from handouts I've created to address problem spots, but they don't cover the whole course, or always do the topics with this sequence of material in mind.
- One goal would be to create my own materials that cover the entirety of the course in this arrangement so that topics flow more naturally one to the other, to avoid the necessity of bouncing around a textbook.
- Some adjustments were made the most recent time I taught the course, some necessitated by a change in textbook, some by a need to improve flow of material.
- More data is needed to determine if my initial results were significant.

REFERENCES

- Online Version of Larson & Edwards Table of Contents, Calculus: Early Transcendental Functions, 5th edition, http://www.webassign.net/features/textbooks/larsonet5/details.html?toc=1&l=search