**Comments on the remarks of President Summers of Harvard that women are innately inferior in mathematics--a discussion of why his interpretation of the statistics is flawed.**

Let's just go once more around about President Summers' (of Harvard) remarks about women and their innate inferiority to men on the high end of intelligence. I have read numerous places online where others have attempted to show why the numbers Summers quotes are bad science. For instance, the fact that he dismisses social factors, social factors which have not been ruled out in creating the IQ numbers he relies on in the first place. This is apparently not enough for his supporters who, like him, are clearly projecting their own bigotry onto the numbers. They see something they think they like and so they are going to stick with it. However, I want to approach this from a different standpoint. All my personal experiences aside, Summers is misinterpreting the mathematics behind the very numbers he's quoting. So let me, a woman mathematician, correct the record and interpret those numbers for him properly.

Let us begin with bell curves. Bell curves are a common **approximation** used in statistical analysis. Statisticians do this for a number of reasons. For one thing, it can be shown that certain secondary phenomena associated with statistics, like the mean of means of samples, can be mathematically shown to fall in bell curves regardless of the original distribution of the data. This makes bell curves an ideal shape for **approximating** the distribution of things. And many things in nature fall into a shape which **approximates** a bell curve. It is the catch-all distribution. It is sometimes used even when the distribution is not exactly bell shaped because bell curves have the convenient property that they can be described by just two numbers: their mean and standard deviation. Other distributions require medians (which are harder to calculate than means) and degrees of freedom, and other details. Bell curves are easy, and so, again, they are frequently used as **approximations** even when it is not entirely appropriate. Others have gone on long discourses about the weaknesses of using bell curves. You can find a discussion of its critics here: http://www.mugu.com/cgi-bin/Upstream/People/Murray/bc-crit.html. We will return to another reason why bell curves may not be a good approximation again below.

Having set out this cautionary tale about bell curves, let us now examine some of the details of Summers' claims. He has quoted research which claims very roughly, that the means of IQs of men and women are the same, but their standard deviations are different, with men have a wider standard deviation. He is pointing to distributions like those shown here: http://www.menweb.org/graphics/bellcurv.gif. The higher peak would represent women, and the lower, more spread out one, men. Let us suppose for a moment that the linked photo is correct in all its details except the scale, which is clearly exaggerated for effect. Summers is using these numbers to suggest that at the very high end of the scale of intelligence, that there are more men who are highly gifted--and better qualified to be in academics--than there are women. Let us suppose for a moment that we accept his numbers to be true, that men and women have the same mean intelligence, but that they have different standard deviations. Let me address rather the problems with his interpretation of these numbers.

Let me remind the reader of some other well-known facts that generated these numbers, that there are more men who have extremely low intelligence and more men who are prodigies. Let us examine the character of both of these high and low end intelligences on the male scale.

For the low end intelligence, the source of these cases is often, in fact, Down's Syndrome. Down's Syndrome is a genetic defect which is known to disproportionately affect men rather than women. The source of the defective gene is on the X chromosome. Since women have two, one defective gene will

not make them suffer this disease, but men have only the one X chromosome, so there is no second gene to pick up where the first falls short. One's parents do not have to have the disease to contract it, and in fact, it can appear 'out of nowhere' having been recessive for many generations. One has to ask oneself if Down's Syndrome cases do not in fact create a deviation from the standard bell curve pattern.

Let us also consider the prodigies. Let me first be clear that Mozart would likely qualify, but Einstein probably would not. Why? Because extreme prodigies tend to appear like Downs' Syndrome, spontaneously. Most prodigies' parents are not themselves prodigies. Also, Mozart shows typical prodigy behaviour, being extremely advanced by age 5... whereas Einstein went through school like everyone else and got a degree in physics in the usual way; he just couldn't find a job in academics. I am not suggesting Einstein was not brilliant, but just that the extreme character of his intelligence may not be on the same level as Mozart. These extreme prodigies, whose parents are not themselves extremely intelligent--which I remind again is the rule rather than the exception--could these not likely also be the result of genetic mutation? Would these prodigies also not deviate from the standard bell curve of the rest of the population?

The point I am trying to make here is that it is reasonable to suppose that there are in fact deviations from the rest of the population curve at these extreme ends of the spectrum in the data for men that is not present in the data for women. The deviations at these large distances from the mean **do not need to be many in number** to produce significant differences in the standard deviation of the bell curve **approximation**. However, should these bubbles at either end exist, it will make the **approximation** a far poorer **estimation** of the population, not a better one. Outlying values are well-known to destroy good statistical information by including them where they do not belong. It is for this reason that statisticians often exclude such values. If these extreme outliers (these genetic anomalies) were removed from the data and the standard deviation were recalculated, and IF the standard deviation of men remained significantly larger than that of women--which it should if the value is real--then barring social factors, one could more reasonably make the conclusions Summers is jumping to; however, I believe that if these extreme outliers are removed, then you would see that the differences in standard deviations are VERY SMALL, if not **exactly zero**, in which case, the vast majority of the population would pattern identically, with men having a handful of outliers at the top and bottom of the scale, certainly not enough to justify continuing to claim that women are not as capable of doing mathematics as men.

In returning to the issue of the bell curve itself, it is also clear that the IQ scale cannot actually be a precise bell curve. Consider that with 100 being the mean, there is a clear lower limit on the lowest possible score of 0--in which case you are dead. However, there is no upper limit in the same way. Consider Marilyn vos Savant, who is said to have an IQ of 262, and to be the smartest person alive on earth today. To completely balance her score 162 points above the mean... it's impossible to have a score 162 points below the mean. That would be negative 62. Regardless of how many standard deviations that score represents, it's possible to be very far indeed from the mean above, and not possible to be quite so far below. This suggests that while a bell curve may serve as a useful **approximation** for most purposes, the true distribution must actually be skewed. This, too, affects the standard deviation result and the real behaviour very far from the mean; this destroys the reliability of making generalizations at the tails based on a single number.

As I have mentioned before, and will reiterate here, one does not need an IQ of 200, or to be any kind of prodigy, to be a professor at Harvard. And furthermore, by the time you get down to the level of IQ you actually do need to have to be a professor at a prestigious university, the difference in numbers of men who qualify and women who qualify are not as significant as Summers' own hiring practices would

suggest--where only 3 of 32 people offered tenure were women.

What this leaves us with are the social factors, that women choose careers other than academics. And these social factors have nothing to do with a woman's merit or their ability to do good science. It has everything to do with things that prestigious universities should be wanting to correct. This has nothing to do with affirmative action programs for women. How else can they attract the best minds? That should, after all is said and done, be the point.

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And on last thought for you to chew on... one additional factor that makes dealing with these extreme values of both the severely retarded and exceptionally intelligent that much more difficult: IQ scores at these extreme ends of the scale are nearly impossible to calibrate properly because there is so little data about other test takers at the extremes. Even where I personally sit on the IQ range, I've been told my actual IQ score could be off by as much as 30 points because there was so little data about others where I myself am--and I don't qualify as a prodigy. On the exam in question, I had scored 167, so I might actually have had an IQ around 197, but the score I got was their 'best guess'. (And yes, the largest error would have been in that direction, since by the time you get around a score of 137, there are many more people--one would have been well below a Mensa qualifying score on that exam--and so the test would have been far easier to calibrate.) Because of this, it suggests that outliers of both men and women should be routinely discarded. If the different standard deviations are real, they will show up throughout the data, not just on the ends. Independent and large scale studies of the highly intelligent are needed to clear this up, independent of what the rest of the population is doing, rather than just making overbroad generalizations based on too little data.